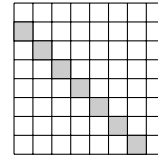
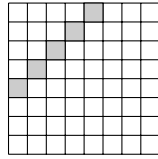
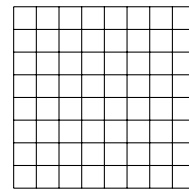
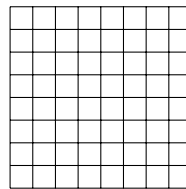
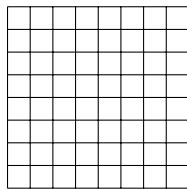
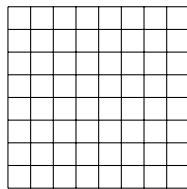


4. [2 parts, 10 points each] In a checkerboard, a *diagonal* is a maximal set of cells whose centers are on a line with a slope of 1 or -1 . Two examples of diagonals follow.



- (a) Show that it is possible to mark 28 cells of the 8×8 checkerboard such that every diagonal contains at most 2 marked cells. Four boards are given below for your convenience. Clearly indicate your final solution.



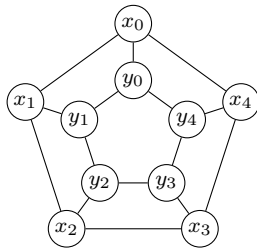
- (b) Prove that no matter how 29 cells are marked, some diagonal contains at least 3 marked cells.

5. [2 parts, 10 points each] Graphs and degrees.

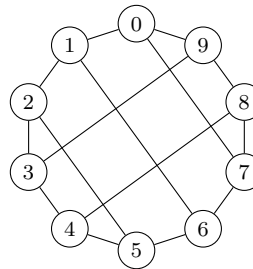
(a) Construct an 8-vertex graph in which one vertex has degree 4 and the rest of the vertices have degree 2.

(b) Prove that there is no 8-vertex bipartite graph in which one vertex has degree 4 and the rest of the vertices have degree 2. (Hint: suppose for a contradiction that G is such a bipartite graph with parts X and Y . What can you say about $\sum_{v \in V(G)} d(v)$ and $\sum_{v \in X} d(v)$?)

6. [10 points] Determine if G and H are isomorphic. If they are isomorphic, then give an isomorphism. If not, then give a property that distinguishes the graphs.



G



H

7. [25 points] Recall that K_3 is the triangle and $K_{1,3}$ is the complete bipartite graph with parts of sizes 1 and 3. Show that $r(K_3, K_{1,3}) = 7$. Be sure to show both that $r(K_3, K_{1,3}) > 6$ and $r(K_3, K_{1,3}) \leq 7$.

