Math378 **Quiz 9** **Wed. Apr. 1, 2026**

Name: Solutions

Directions: Show all work. You may leave your answer in terms of factorials, falling factorials, and binomial coefficients. Any sums or products should be as simple as possible.

1. **[5 parts, 2 points each]** A fair 6-sided die is rolled 3 times.

   (a) Give the sample space Σ.

$$\Sigma = \boxed{\{(x_1, x_2, x_3) : \text{each } x_i \text{ is in } \{1,2,3,4,5,6\}\}} = \boxed{\{1,\dots,6\}^3} = \boxed{[6]^3}.$$

   (b) What is the probability that all 3 rolls give the same value?

$A_1 = \{(x_1,x_2,x_3) \in \Sigma : x_1 = x_2 = x_3\}$. Note $|A_1| = 6$ (choose the value).

So $\Pr(A_1) = \dfrac{|A_1|}{|\Sigma|} = \dfrac{6}{6^3} = \dfrac{1}{6^2} = \boxed{\dfrac{1}{36}}$

   (c) What is the probability that all 3 rolls give distinct values?

Let $A_3 = \{(x_1,x_2,x_3) \in \Sigma : x_1 \neq x_2,\ x_1 \neq x_3,\ x_2 \neq x_3\}$. Note $|A_3| = 6 \cdot 5 \cdot 4$

(2) 5 opts

(1) 6 opts (3) 4 opts

So $\Pr(A_3) = \dfrac{|A_3|}{|\Sigma|} = \dfrac{6\cdot 5\cdot 4}{6^3} = \dfrac{5\cdot 4}{6^2} = \dfrac{20}{36} = \boxed{\dfrac{5}{9}}$

   (d) What is the probability that among the 3 rolls, one of the values is distinct and the other two are equal?

Let A_2 be the event that among the 3 rolls, we see exactly 2 values. Since $\{A_1, A_2, A_3\}$ is a partition of Σ, we have $|A_1| + |A_2| + |A_3| = |\Sigma|$, so $|A_2| = |\Sigma| - |A_1| - |A_3|$ $= 6^3 - 6 - 6\cdot 5\cdot 4 = 6[6^2 - 1 - 20] = 6[36 - 21] = 6\cdot 15$. Hence $\Pr(A_2) = \dfrac{|A_2|}{|\Sigma|} = \dfrac{6\cdot 3\cdot 5}{6^3} = \dfrac{3\cdot 5}{6\cdot 6} = \boxed{\dfrac{5}{12}}$

   (e) What is the probability that at least 1 of the rolls is a 4?

Let $A_4 = \{(x_1,x_2,x_3) \in \Sigma : 4 \in \{x_1,x_2,x_3\}\}$. We count the complement. Now $\overline{A_4} =$ $\{(x_1,x_2,x_3) : x_1 \neq 4,\ x_2 \neq 4, \text{ and } x_3 \neq 4\}$, and so $|\overline{A_4}| = 5^3$

$\{1,2,3,5,6\}$ 5 opts

Therefore $|A_4| = |\Sigma| - |\overline{A_4}| = 6^3 - 5^3$ and

$$\Pr(A_4) = \frac{|A_4|}{|\Sigma|} = \frac{6^3 - 5^3}{6^3} = \frac{216 - 125}{216} = \boxed{\frac{91}{216}}.$$