Name: ______________________________

Directions: Show all work.

1. In a game of Nim, tokens are placed in various piles. In each turn, players removing one or more tokens from a selected pile. In this variant, the game begins with n piles, each starting with 10 tokens. When no tokens remain, the player to move loses.

   (a) [**3 points**] Prove that if n is even, then Player 2 has a winning strategy. (Hint: Analyzing the general game of Nim is tricky; give an explicit strategy for Player 2 in this special case.)

   (b) [**2 points**] Use part (a) to show that if n is odd, then Player 1 has a winning strategy.

2. **[5 points]** Let $a_0 = 0$ and $a_n = \frac{1}{2-a_{n-1}}$ for $n \geq 1$. Guess a formula for a_n and prove your formula is correct.