

Name: _____

Directions: Show all work.

1. [6 points] For $n \geq 0$, let

$$S_n = \sum_{k=1}^{2n} (-1)^k k^2 = -1^2 + 2^2 - 3^2 + 4^2 - \dots - (2n-1)^2 + (2n)^2.$$

Prove that for each non-negative integer n , we have that $S_n = (2n+1)n$.

2. [**2 parts, 2 points each**] Recall that the adjusted Fibonacci sequence is defined by $\hat{F}_0 = \hat{F}_1 = 1$ and $\hat{F}_n = \hat{F}_{n-1} + \hat{F}_{n-2}$ for $n \geq 2$.
- (a) Let n be a positive integer and let t be the maximum integer such that $\hat{F}_t \leq n$. Prove that $n - \hat{F}_t < \hat{F}_{t-1}$. (Hint: there is a short proof, without induction/no minimum counter-example. If stuck, then try a proof by contradiction.)
- (b) Prove that each positive integer n is a sum of distinct, non-consecutive adjusted Fibonacci numbers. For example, if $n = 20$, then we have $20 = 13 + 5 + 2 = \hat{F}_6 + \hat{F}_4 + \hat{F}_2$.