

Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [3.1.11] How many numbers in $\{1000, \dots, 9999\}$ are divisible by 5 and contain the digit 4?
2. You choose a positive integer less than or equal to 10,000 at random. What is the probability that your chosen integer:
 - (a) has all distinct digits?
 - (b) [3.2.4] has exactly one 4 and one 7?
 - (c) has at least one 4 and at least one 7? (Hints: Count the complement. Let $U = \{0000, \dots, 9999\}$, let A be the set of $n \in U$ that have no 4, and let B be the set of $n \in U$ that have no 7. Use that $|A \cup B| = |A| + |B| - |A \cap B|$.)
3. A fair coin is tossed 9 times. Find the probability that the flips form a palindromic sequence (e.g. HHTTHTTTH).
4. [3.2.8] Ten soccer players are standing in a circle and randomly passing a ball. When a player gets the ball, they can pass the ball to anyone except to the player who just passed them the ball.
 - (a) If player A starts the exercise, what is the probability that A receives the third pass?
 - (b) If player A does not start the exercise, what is the probability that A receives the third pass?
5. [4.1.3] A standard deck of playing cards has one card for each suit/rank pair, where the 4 suits are spades, hearts, diamonds, and clubs and the 13 ranks are ace, 2 through 10, jack, queen, king. How many ways are there to order a deck of cards so that all cards with the same suit are next to each other? (The cards within each suit need not be in order.)
6. [4.1.10] You order 10 different books online, 3 of which are for your sister. The books arrive randomly, one by one. What is the probability that the books for your sister arrive consecutively?