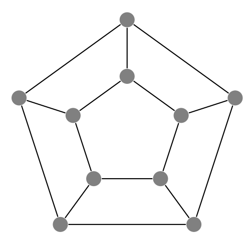
Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. Graphs, Subgraphs, and Isomorphism.

   (a) Let G be a graph. The *neighborhood* of a vertex $v \in V(G)$, denoted $N(v)$, is the set of all vertices adjacent to v. Show that the 4-cycle C_4 is not a subgraph of G if and only if for all distinct $u, v \in V(G)$, we have $|N(u) \cap N(v)| \leq 1$. Note: a vertex w in $N(u) \cap N(v)$ is a *common neighbor* of u and v.

   (b) Recall that the Petersen graph is the graph whose vertices are the 2-element subsets of $\{1, 2, 3, 4, 5\}$ where u and v are adjacent if and only if $u \cap v = \varnothing$. Use part (a) to show that the Petersen graph does not contain a 4-cycle.

   (c) Use part (b) to give a short proof that the graph H below is *not* isomorphic to the Petersen graph.



2. [2.3.2] Suppose that $r(4, 5) = 25$. Let G be a copy of K_{25} in which each edge is colored red or blue. Prove that G either contains a monochromatic copy of K_5 (red or blue), or G contains both a red copy and a blue copy of K_4.

3. [2.3.8] Prove that $r(3, 4) > 8$.