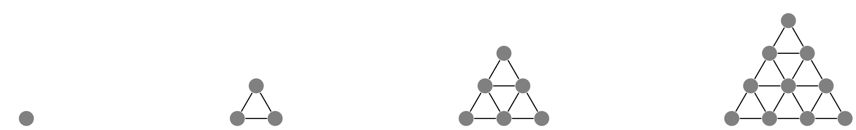
Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. **[15 points]** Let $T_1, \ldots, T_n$ be a list of domino tilings of a (2×8)-grid. (Note that each entry in the list is a complete tiling, so for example T_1 might be the tiling that places all n dominos vertically.) What is the minimum n such that two tilings on the list must be identical?

2. **[2 parts, 5 points each]** The *triangular lattice* G_n is the graph whose vertices are arranged in rows of sizes $1, 2, \ldots, n$, with the midpoints of the rows centered on a common vertical line. Consecutive vertices in the same row are adjacent, and the jth vertex in row i is adjacent to the jth and $(j+1)$st vertex in row $i+1$. No other pairs of vertices are adjacent. See below.



G_1 G_2 G_3 G_4

Note that G_n has $\binom{n+1}{2}$ vertices.

(a) Find a formula for the number of edges in G_n.

(b) Let d_n be the average of the degrees of vertices in G_n. Find a formula for d_n. What is $\lim_{n\to\infty} d_n$? Does this make sense?

3. Let n be a positive integer and suppose that $A \subseteq \{1, 2, \ldots, 5n\}$.

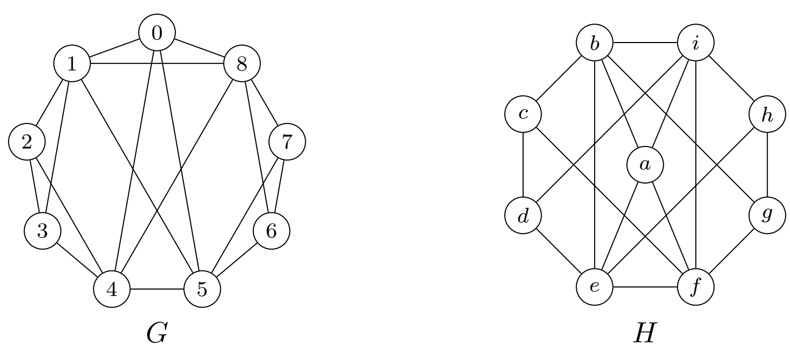
   (a) **[15 points]** Show that if $|A| > 2n$, then there exists $x, y \in A$ such that $y - x = 2$ or $y - x = 3$. (Hint: partition $\{1, \ldots, 5n\}$ into n intervals, each of size 5.)

   (b) **[10 points]** Show that if $|A| = 2n$, then the conclusion in part (a) need not hold.

4. **[10 points]** Let n be a positive integer. Prove that there exists a $2n$-vertex graph with n vertices of degree n and n vertices of degree $n+1$ if and only if n is even.

5. **[5 points]** Give the definition of a *bipartite graph*.

6. **[10 points]** Are the following graphs isomorphic? Either give an isomorphism or explain why not.



7. **[25 points]** Recall that P_3 is the path on 3 vertices. Show that $r(P_3, K_5) = 9$. Be sure to show both that $r(P_3, K_5) > 8$ and $r(P_3, K_5) \leq 9$.