Name: _____

Directions: Show all work.

1. [**5 points**] Let $n \geq 1$. Give the identity that is proved by the following combinatorial argument: Each non-empty subset of $\{1, \ldots, n\}$ consists of a maximum element k for some $1 \leq k \leq n$ together with a subset of $\{1, \ldots, k-1\}$.

2. [**5 points**] Let $n \geq 1$. Give a combinatorial proof that $n! - 1 = \sum_{k=1}^{n}(k-1)(k-1)!$. (Hint: except for the identity permutation σ where $\sigma = 12 \cdots n$, every permutation σ of $[n]$ has at least one index i such that the value of σ at position i is not equal to i.)