

Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

- [5.2.4] Find a simple expression for $\sum_{k=0}^n (2k+1) \binom{n}{k}$.
- Let $n \geq 1$, let O_n be the set of odd numbers in $\{0, 1, \dots, n\}$, and let E_n be the set of even numbers in $\{0, 1, \dots, n\}$. Give a combinatorial proof that $\sum_{k \in E_n} \binom{n}{k} = \sum_{k \in O_n} \binom{n}{k}$.
- [5.2.9] Let $n \geq 1$, let O_n be the set of odd numbers in $\{0, 1, \dots, n\}$, and let E_n be the set of even numbers in $\{0, 1, \dots, n\}$. Let $a_n = \sum_{k \in E_n} k \binom{n}{k}$ and $b_n = \sum_{k \in O_n} k \binom{n}{k}$.
 - Use the binomial theorem to find expressions for $a_n + b_n$ and $a_n - b_n$.
 - Find formulas for a_n and b_n .
- [8.1.3] How many permutations of the letters in SCRIPPS have no two consecutive letters the same?
- How many permutations of the letters in AABBC...ZZ have no two consecutive letters the same? Find a summation formula.
- [10.1.2] Is 7, 7, 6, 5, 4, 4, 4, 3, 2 a graphic sequence?