Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. How many ways are there to arrange the letters of MISSISSIPPI:
   (a) with no additional restrictions?
   (b) [4.3.7] if all four S's cannot appear consecutively?
   (c) if no two S's can appear consecutively?
2. [4.4.2] I want to buy exactly 10 jars of various herbs and spices, and I am only interested in Cinnamon, Curry, Cumin, Caraway, Coriander, and Chervil. The supermarket has plenty of each. How many different combinations are possible?
3. [4.4.{8-11}] Solutions to equations.
   (a) Count the integral solutions to $x_1 + x_2 + x_3 + x_4 = 30$ with $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$, and $x_4 \geq 8$.
   (b) Count the integral solutions to $x_1 + \cdots + x_5 = 47$ with $5 \leq x_i \leq 30$ for each i.
   (c) How many non-negative integer solutions are there to $x_1 + \cdots + x_8 = 47$, where exactly three of the variables are equal to zero? What if we wanted at least three variables equal to zero?
   (d) Find the number of non-negative integer solutions to $x_1 + \cdots + x_7 \leq 47$.
4. How many ways are there to form a subset of $[n]$ of size k with the property that each selected number is at distance at least 3 from every other selected number? For example, if $n = 8$ and $k = 3$ there are 4 ways: $\{1, 4, 7\}$, $\{1, 4, 8\}$, $\{1, 5, 8\}$, and $\{2, 5, 8\}$.
5. Consider a fair die with t sides, labeled 1 through t.
   (a) What is the probability that in n rolls of the die, the same result does not appear twice consecutively?
   (b) Let X be the random variable that equals the number of times the die is rolled when we first get consecutive rolls with the same value. Let X_j be the random variable which equals 1 if the rolls 1 through j have no consecutive rolls with the same result, and 0 otherwise. Explain why $X = 1 + \sum_{j \geq 1} X_j$.
   (c) Compute the average number of times the die must be rolled to see the same result twice in a row. (Hint: recall the geometric series formula $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ for $|z| < 1$.)