Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 parts, 1 point each]** Let $A = \{\varnothing, (1,2)\}$ and $B = \{\varnothing, (2,1)\}$. Find the following sets.

   (a) $A \times B$

   $A \times B = \boxed{\{(\varnothing, \varnothing), (\varnothing, (2,1)), ((1,2), \varnothing), ((1,2), (2,1))\}}$

   (b) $A \times A \times B$

   $A \times A \times B = \boxed{\{(\varnothing, \varnothing, \varnothing), (\varnothing, \varnothing, (2,1)), (\varnothing, (1,2), \varnothing), (\varnothing, (1,2), (2,1)), ((1,2), \varnothing, \varnothing), ((1,2), \varnothing, (2,1)), ((1,2), (1,2), \varnothing), ((1,2), (1,2), (2,1))\}}$

2. **[1 point]** Give an example of an element in $\mathbb{R}^2 \times \mathbb{R}^3 \times \mathbb{R}$.

   $\boxed{((1,2), (2,3,4), 5)}$

3. **[3 parts, 1 point each]** Decide whether the following statements are true or false. Write the entire word true or the entire word false to indicate your answer. No explanations or justification required.

   (a) $\{1, 2, 3\} \in \{1, 2, 3, 4\}$ $\boxed{\text{FALSE}}$ $(\{1,2,3\} \subseteq \{1,2,3,4\})$.

   (b) $\mathbb{Q} \subseteq \{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$ $\boxed{\text{FALSE}}$: $\frac{1}{2} \in \mathbb{Q}$ but $\frac{1}{2} \notin \{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

   (c) $\{\mathbb{Q}\} \subseteq \{\mathbb{R}\}$ $\boxed{\text{FALSE}}$: $\mathbb{Q} \in \{\mathbb{Q}\}$ but $\mathbb{Q} \notin \{\mathbb{R}\}$.

4. **[2 parts, 1 point each]** Find the following power sets.

   (a) $\mathcal{P}(\{a, b, c\})$

$$\mathcal{P}(\{a,b,c\}) = \boxed{\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}}$$

   (b) $\mathcal{P}(\{\overset{a}{\varnothing}, \overset{b}{\overbrace{(1, 2)}}\})$

$$= \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$$

$$= \boxed{\{\emptyset, \{\emptyset\}, \{(1,2)\}, \{\emptyset, (1,2)\}\}}$$

5. **[1 point]** Let A and B be sets, and suppose that $\mathcal{P}(A) \times B = \varnothing$. What, if anything, can we conclude about A and B? Explain.

For $\mathcal{P}(A) \times B = \emptyset$ to hold, either $\mathcal{P}(A)$ or B is non-empty.

Since $\emptyset \in \mathcal{P}(A)$ for every set A, a power set is never empty.

Therefore $\boxed{B = \emptyset}$ but $\boxed{\text{A could be any set.}}$

6. **[1 point]** Let $A = \{1, 2, 3, 4\}$. Express the set $\{X \subseteq A \colon |X| \in \{2, 3\}\}$ by listing its elements between braces.

This set contains the subsets of A that have cardinality 2 or 3.

So the desired set is

$$\boxed{\{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}}$$