Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. See "Guidelines and advice' on the course webpage for more information.

1. Translate the following English statements to symbolic logic. Is the statement true or false? Explain.
   (a) Every set has a subset.
   (b) Every nonempty set has a subset.
   (c) Every set has a nonempty subset.
   (d) Every nonempty set has a nonempty subset.
   (e) An integer plus an integer always equals an integer.
   (f) Every nonempty subset of the natural numbers has a maximum integer.
   (g) Between every pair of distinct rational numbers, there is a third rational number.
   (h) There is a rational number which is between every pair of distinct rational numbers.

2. Translate the following sentences in symbolic logic to English. Is the statement true or false? Explain.
   (a) $\exists x \in \mathbb{Z}, x^4 = 81$
   (b) $\forall x \in \mathbb{Z}, x^2 \geq 0$
   (c) $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x + y > x \land x + y > y$
   (d) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, (x = 2y) \lor (x = 2y + 1)$
   (e) $\forall x \in \mathbb{Z}, [(\exists y \in \mathbb{Z}, x = 6y) \iff (\exists y \in \mathbb{Z}, x = 2y) \land (\exists y \in \mathbb{Z}, x = 3y)]$.

3. Carefully negate the following sentences as simply and naturally as possible.
   (a) The integer 9 is odd and a perfect square.
   (b) If 8 is even, then 8 is a perfect square.
   (c) A real number x is rational if and only if $2x$ is rational.
   (d) Some integer is both even and odd.
   (e) Every integer is even or odd, but not both.
   (f) All values of the function $\cos(x)$ are bounded between -1 and 1.
   (g) For every positive real number ε, there exists a positive real number δ such that for all $x \in \mathbb{R}$, if $|x - 2| < \delta$ then $|\ln x - \ln 2| < \varepsilon$. [*Hints*: it may be easier to first translate to symbolic logic, then negate and simplify, and then translate back. It will be convenient to define $\mathbb{R}^+ = \{x \in \mathbb{R} \colon x > 0\}$, so that the translation to symbolic logic can begin $\forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \ldots$.]

4. *Balanced statements.* Using only the logical operands $\land$ and $\lor$, write a statement φ which is true if and only if at least two of $\{P_1, \ldots, P_8\}$ are true, while using each P_j at most 3 times. For example,

$$[P_1 \land (P_2 \lor \ldots \lor P_8)] \lor [P_2 \land (P_3 \lor \ldots \lor P_8)] \lor \ldots \lor [P_7 \land P_8]$$

   is logically equivalent to the desired sentence φ, but it is not balanced since P_8 is used a total of 7 times.