

Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. See “Guidelines and advice” on the course webpage for more information.

- For each $n \in \mathbb{N}$, let $A_n = \{2n, 2n + 1, \dots, 3n\}$.
 - Find $\bigcup_{z=15}^{20} A_z$ and $\bigcap_{s=15}^{20} A_s$.
 - Suppose that $a, b \in \mathbb{N}$ and $a \leq b$. Find $|\bigcup_{k=a}^b A_k|$ in terms of a and b .
- [BP 1.8.{6,8,10}] Recall that for real numbers a and b , we have $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$. Find the following sets.

<ol style="list-style-type: none"> $\bigcup_{i \in \mathbb{N}} [0, i + 1]$ $\bigcap_{i \in \mathbb{N}} [0, i + 1]$ $\bigcup_{\alpha \in \mathbb{R}} (\{\alpha\} \times [0, 1])$ 	<ol style="list-style-type: none"> $\bigcap_{\alpha \in \mathbb{R}} (\{\alpha\} \times [0, 1])$ $\bigcup_{x \in [0, 1]} ([x, 1] \times [0, x^2])$ $\bigcap_{x \in [0, 1]} ([x, 1] \times [0, x^2])$
---	---
- For each $k \in \mathbb{N}$, let $A_k = \{kn : n \in \mathbb{Z}\}$. Find the following sets.

<ol style="list-style-type: none"> The examples A_1, A_2, and A_3. $\bigcup_{k=1}^3 A_k$ $\bigcup_{k=2}^4 A_k$ 	<ol style="list-style-type: none"> $\bigcap_{k=1}^3 A_k$ $\bigcap_{k=1}^{\infty} A_k$ $\bigcup_{k \in I} A_k$ where $I = \{3, 5, 7, 9, 11, \dots\}$.
--	---
- [BP 2.1, evens] Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.
 - Every even integer is a real number.
 - Sets \mathbb{Z} and \mathbb{N} .
 - Some sets are finite.
 - $\mathbb{N} \notin \mathcal{P}(\mathbb{N})$.
 - $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$.
 - If the integer x is a multiple of 7, then it is divisible by 7.
 - Call me Ishmael.
 - If x is an integer, then $x + y$ is also an integer.
- An infinite series of nested circles and squares are drawn, all sharing a common center point. The outermost circle has radius 1. The space between each circle and the square it circumscribes is shaded. What is the total area of the shaded regions?

