Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. How many ways are there to arrange the letters of MISSISSIPPI:
   (a) with no additional restrictions?
   (b) [4.3.7] if all four S's cannot appear consecutively?
   (c) if no two S's can appear consecutively?

2. [4.4.2] I want to buy exactly 10 jars of various herbs and spices, and I am only interested in Cinnamon, Curry, Cumin, Caraway, Coriander, and Chervil. The supermarket has plenty of each. How many different combinations are possible?

3. [4.4.{8-11}] Solutions to equations.
   (a) Count the integral solutions to $x_1 + x_2 + x_3 + x_4 = 30$ with $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$, and $x_4 \geq 8$.
   (b) Count the integral solutions to $x_1 + \cdots + x_5 = 47$ with $5 \leq x_i \leq 30$ for each i.
   (c) How many non-negative integer solutions are there to $x_1 + \cdots + x_8 = 47$, where exactly three of the variables are equal to zero? What if we wanted at least three variables equal to zero?
   (d) Find the number of non-negative integer solutions to $x_1 + \cdots + x_7 \leq 47$.

4. How many ways are there to form a subset of $[n]$ of size k with the property that each selected number is at distance at least 3 from every other selected number? For example, if $n = 8$ and $k = 3$ there are 4 ways: $\{1, 4, 7\}$, $\{1, 4, 8\}$, $\{1, 5, 8\}$, and $\{2, 5, 8\}$.

5. [5.1.5] Let $c \leq b \leq a$ be non-negative integers. Give two proofs, one combinatorial, for $\binom{a}{b}\binom{b}{c} = \binom{a}{c}\binom{a-c}{b-c}$.