

Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [3.1.6] How many odd five-digit integers start with an even digit?
2. [3.1.8] Count the functions $f: \{1, \dots, 7\} \rightarrow \{1, 2, 3, 4\}$.
3. [3.1.14] How many ways are there to color the vertices of a pentagon with three colors such that no two adjacent vertices receive the same color?
4. [3.2.3] You roll a regular six-sided die twice.
 - (a) What is the probability that you roll five twice?
 - (b) If you roll a five the first time, then what is the probability that you also roll a five on the second try?
 - (c) If all we know is that at least one of your rolls was a five, then what is the probability that you rolled five twice?
5. Suppose each integer in $[n]$ is colored red, yellow, or green at random.
 - (a) What is the probability that no consecutive integers get the same color?
 - (b) Let a_n be the number of colorings where red never immediately follows green. For example, $a_0 = 1$ (since the null coloring counts), $a_1 = 3$, and $a_2 = 9 - 1 = 8$ (since 'GR' is the only disallowed coloring). Prove that $a_n = 3a_{n-1} - a_{n-2}$ for $n \geq 2$.
 - (c) Find a recurrence for p_n , the probability that red never immediately follows green. What is the smallest n for which this probability is less than $1/2$?