Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See "Guidelines and advice" on the course webpage for more information.

1. Use induction to prove the following.
   (a) For each $n \in \mathbb{N}$, we have $\sum_{i=1}^{n}(8i-5) = 4n^2 - n$.
   (b) For each non-negative integer n, we have $9 \mid 4^{3n} + 8$.
   (c) If $n \in \mathbb{N}$, then $2^n + 3^n \leq 5^n$.
   (d) If $n \in \mathbb{N}$, then $(\sum_{k=1}^{n} k)^2 = \sum_{j=1}^{n} j^3$.
   (e) If $n \in \mathbb{N}$, then $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$. (Hint: you may need one to use one of our identities involving binomial coefficients.)

2. Using any method, prove the following.
   (a) If $x_1, \ldots, x_n$ are non-negative real numbers, then
   $$(1+x_1)(1+x_2)\cdots(1+x_n) \geq 1 + x_1 + x_2 + \cdots + x_n.$$
   (b) If $n \in \mathbb{Z}$, then $\gcd(3n+5, 5n+8) = 1$.
   (c) If n is a non-negative integer, then $(1+\sqrt{2})^n + (1-\sqrt{2})^n$ is an even integer.

3. Recall the Fibonacci numbers, defined by $F_1 = 1$, $F_2 = 1$, and $F_k = F_{k-1} + F_{k-2}$ for $k \geq 2$. Prove that each integer n can be represented as the sum of Fibonacci numbers, no two of which are consecutive. (For example, $2 = F_3$, $10 = F_6 + F_3 = 8+2$, and $20 = F_7 + F_5 + F_3 = 13+5+2$. However, even though $20 = F_7 + F_5 + F_2 + F_1$, this is not a desired representation for 20 since it uses the consecutive Fibonacci numbers F_1 and F_2.)