

Name: Key

1. [6 parts; 1/2 point each] Let $A = \{1, 2, \{3\}, \{4, 5\}, 3\}$, $B = \{\{4, 5\}, 2\}$, and $C = \{\emptyset, \{3\}\}$.

- (a) True or false: $4 \in B$. FALSE (c) True or false: $\emptyset \in C$. True
 (b) True or false: $B \subseteq A$. True (d) True or false: $\{3\} \subseteq C$. False
 (e) Find $A \cap C$. (Use set notation. Give an explicit list of the members.)

$$A \cap C = \{\{3\}\}$$

- (f) Find $C \times B$. (Use set notation. Give an explicit list of the members.)

$$C \times B = \{(\emptyset, \{4, 5\}), (\emptyset, 2), (\{3\}, \{4, 5\}), (\{3\}, 2)\}$$

2. [1 point] Let $\mathcal{P}(S)$ denote the power set of S . For $S = \{a\}$, find $\mathcal{P}(\mathcal{P}(S))$.

$$\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$$

$$\mathcal{P}(\{\emptyset, \{a\}\}) = \{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\emptyset, \{a\}\}\}$$

3. [2 points] Let A and B be infinite denumerable sets. Explain why $A \cup B$ is denumerable.

Since A is denumerable, we can list the elements of A as $a_1, a_2, a_3, \dots$. Similarly, we can list the elements of B as $b_1, b_2, b_3, \dots$.

Alternating between A and B , we can list the elements of $A \cup B$:

$a_1, b_1, a_2, b_2, a_3, b_3, \dots$

Show your work; don't just write down a number

4. [2 parts, 1 point each] An ice-cream parlor has 8 flavors. You plan to buy one scoop of ice-cream each day for the next 6 days.

(a) How many ways are there to select the flavors?

$$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 8^6 = \boxed{262,144}$$

(b) How many ways are there to select the flavors if you do not want to try any flavor more than once?

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = \boxed{20,160}$$

5. [1 point] A palindrome is a string of characters that reads the same forward and backward. How many five-letter palindromes are there using the standard English alphabet $\{a, b, \dots, z\}$? (Note that non-words, such as "krrrk" count.)

$$\begin{array}{c} 26 \cdot 26 \cdot 26 = \boxed{17,576} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{1st and 5th} & \text{2nd and 4th} & \text{5th} \end{array} \end{array}$$

6. [1 point] An ATM pin number is a sequence of 4 digits. How many pin numbers contain at least one even digit and at least one odd digit? For example, the pins 0324 and 5526 count, but 0000, 2884, and 3179 do not.

$$\text{All even: } 5 \cdot 5 \cdot 5 \cdot 5 = 625$$

$$\text{All odd: } 5 \cdot 5 \cdot 5 \cdot 5 = 625$$

$$\text{Total \# pins: } 10 \cdot 10 \cdot 10 \cdot 10 = 10,000$$

$$\begin{aligned} \#(\text{with even and odd}) &= 10,000 - \#(\text{all even}) - \#(\text{all odd}) \\ &= 10,000 - 625 - 625 = \boxed{8750} \end{aligned}$$