Name: Key

1. [**2 points**] The following is pseudocode for a program that takes a non-negative integer x as input and outputs $x!$. Find the loop invariant Q.

```
Factorial(x):
  i = 2
  j = 1
  while i ≠ x + 1 do
      j = j * i
      i = i + 1
  end while

  //j now has the value x!
  return j
```

Q: $j = (i-1)!$

2. [**2 points**] Use the Euclidian algorithm to find gcd(2622, 627). Show the intermediate steps of the Euclidian algorithm; no credit for answers that do not use the Euclidian algorithm.

$$627 \overline{\smash{)}2622} = 4, \quad 2622 - 2508 = 114 \Rightarrow \gcd(627, 114)$$

$$114 \overline{\smash{)}627} = 5, \quad 627 - 570 = 57 \Rightarrow \gcd(114, 57)$$

$$57 \overline{\smash{)}114} = 2, \quad 114 - 114 = 0 \Rightarrow \gcd(57, 0)$$

$$\gcd(57,0) = \boxed{57}$$

3. **[2 points]** Write the first 4 values of the sequence given by $A(1) = 3$, $A(2) = -1$, and $A(n) = 2A(n-1) + A(n-2)$.

$A(1) = 3$
$A(2) = -1$
$A(3) = 2(-1) + 3 = 1$
$A(4) = 2(1) + (-1) = 1$

So $\boxed{3, -1, 1, 1}$.

4. **[2 parts, 1 point each]** A collection S of strings is defined recursively by

   1. The empty string λ belongs to S.
   2. The strings a and b belong to S.
   3. If X belongs to S, then aXa and bXb belong to S.

   (a) Write down three (3) different strings of length 4 that are in S.

   Any 3 of aaaa, abba, baab, bbbb.

   (b) Give a simple, non-recursive definition of S that is equivalent to the given definition.

   S is the set of all palindromes; i.e. S is the set of strings that read the same forward and backward.

5. **[2 points]** Give a recursive definition of x^R, the reverse of the string x.

$$x^R = \begin{cases} \lambda & x = \lambda \\ y^R \alpha & x = \alpha y \end{cases}$$
, where α is a single character and y is a string

OR:

$$x^R = \begin{cases} \lambda & x = \lambda \\ z^R y^R & x = yz \end{cases}$$