Name: Key

1. **[2.5 points]** Verify the correctness of the following program.

$$\begin{array}{|l|}\hline \{\} \\ \quad y = x - 1 \\ \quad y = y * x \\ \{y = x(x-1)\} \\ \hline \end{array}$$

1. $\{\ \}$
$\{(x-1)x = x(x-1)\}$ Logical implication

2. $y = x - 1$ Assignment

$\{y * x = x(x-1)\}$

3. $y = y * x$ Assignment

$\{y = x(x-1)\}$

2. **[2.5 points]** Prove that the sum of two odd integers is even.

Proof: Let x and y be odd integers. Since x and y are odd, $x = 2m+1$ and $y = 2n+1$ for some integers m and n. Hence $x+y = (2m+1)+(2n+1) = 2(m+n+1)$. Since $m+n+1$ is an integer and $x+y$ is twice this value, $x+y$ is even. ∎

3. **[2.5 points]** Prove that $x(x+1)$ is even for every integer x.

Proof: We consider two cases.

Case 1: x is even. Since x is even, $x=2m$ for some integer m. Hence $x(x+1) = 2m(x+1)$.

Case 2: x is odd. Since x is odd, $x=2m+1$ for some integer m. Hence $x(x+1) = x((2m+1)+1) = x(2m+2)$ $= 2x(m+1)$.

In both cases, $x(x+1)$ is twice an integer and is therefore even. ▨

4. **[2.5 points]** Using the provided lemma, prove that $\sqrt{2}$ is not a rational number.

Lemma 1. *The product xy is odd if and only if x is odd and y is odd.*

Proof: By contradiction. Suppose that $\sqrt{2}$ is rational, so that $\sqrt{2} = \frac{p}{q}$ for some integers p and q that have no common factors except ± 1. Hence $\sqrt{2}q = p$, and squaring both sides gives $2q^2 = p^2$. Therefore p^2 is even. Since $p \cdot p$ is even, the lemma implies that p is even. Since p is even, $p = 2m$ for some integer m. But then $2q^2 = (2m)^2 = 4m^2$, and so $q^2 = 2m^2$, which implies that q^2 is even. By the lemma, q is even. Therefore both p and q are divisible by 2, a contradiction.

▨