

Name: _____

Unless told otherwise, show your work. Answers without work earn reduced credit.

1. [3 points] Decide whether the given functions are one-to-one/injective, onto/surjective, or bijective. For each blank cell in the table, write “Yes” if the function has the property, and “No” otherwise. You do not need to show your work.

In the following, let A^* be the set of finite strings of a 's and b 's. For example, $aaba$, bb , and the empty string λ are all in A^* . Recall that $\mathbb{N} = \{0, 1, 2, \dots\}$ and \mathbb{Z} is the set of integers.

Function	one-to-one	onto	bijective
$f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x) = x - 1$			
$f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x) = x^2 - 1$			
$f: \mathbb{Z} \rightarrow \mathbb{Z}$ where $f(x) = x^3 - 1$			
$f: A^* \rightarrow \mathbb{N}$ where $f(x)$ equals the length of x			
$f: A^* \rightarrow A^*$ where $f(x) = xx$			
$f: A^* \rightarrow A^*$ where $f(x)$ equals the reverse of x			

2. [2 parts, 1 point each] Let A be the set of all strings of a 's and b 's of length 8. Let $f: A \rightarrow A$ be the function that shifts every character to the right, and moves the 8th character to the front of the string. For example, $f(abaaabbb) = babaabbb$. Let $g: A \rightarrow A$ be the function that reverses the string. For example, $g(babaabbb) = bbaaabab$.

(a) Find $(f \circ g)(abbababb)$.

(b) Let $h = f \circ g$. Is h a bijection? If h is a bijection, describe the inverse h^{-1} . If h is not a bijection, explain why.

3. [3 parts, 1 point each] Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Express the following permutations as the composition of zero or more disjoint cycles; each cycle should have at least 2 elements.

(a) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 2 & 6 & 1 & 4 & 3 & 8 & 7 \end{pmatrix}$

(b) $(3\ 6\ 8\ 4) \circ (6\ 2\ 4\ 5)$

(c) $(2\ 6\ 8) \circ (2\ 7) \circ (3\ 1\ 6\ 5) \circ (4\ 2)$

4. [2 points] Prove, by finding constants that satisfy the definition of order of magnitude, that $f = \Theta(g)$ if $f(n) = 3 \log(n^5)$ and $g(n) = \log(n)$.