Name: Answer Key

1. [**3 parts, 1 point each**] Write the negation of the following sentences.

   (a) The weather is hot and dry.

   The weather is cold or wet.

   (b) Carl is short or strong.

   Carl is tall and weak.

   (c) Either it will rain or it will snow, but not both.

   Either there will be no rain and no snow or it will both rain and snow. Also OK: It will rain if and only if it will snow.

2. Two parts.

   (a) [**3 points**] Write a truth table for the following wff:

$$(A \leftrightarrow B) \wedge (B \vee C) \wedge (A \to C) \to C$$

A	B	C	P_1: $A \leftrightarrow B$	P_2: $B \vee C$	P_3: $A \to C$	$P_1 \wedge P_2 \wedge P_3$	$P_1 \wedge P_2 \wedge P_3 \to C$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	T	F	T
F	T	F	F	T	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	F	T	F	T

   (b) [**1 point**] Is the wff a tautology? Briefly explain why or why not.

   Yes, since the wff is true regardless of the truth values of A, B, and C.

Derivation Rule	Name/Abbreviation for Rule
$P \vee Q \iff Q \vee P$ $P \wedge Q \iff Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R \iff P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)' \iff P' \wedge Q'$ $(P \wedge Q)' \iff P' \vee Q'$	De Morgan's laws—De Morgan
$P \to Q \iff P' \vee Q$	Implication—imp
$P \iff (P')'$	Double negation—dn
$P \leftrightarrow Q \iff (P \to Q) \wedge (Q \to P)$	Defn of Equivalence—equ
$\left.\begin{array}{c} P \\ P \to Q \end{array}\right\} \implies Q$	Modus ponens—mp
$\left.\begin{array}{c} P \to Q \\ Q' \end{array}\right\} \implies P'$	Modus tollens—mt
$\left.\begin{array}{c} P \\ Q \end{array}\right\} \implies P \wedge Q$	Conjunction—con
$P \wedge Q \implies \left\{\begin{array}{c} P \\ Q \end{array}\right.$	Simplification—sim
$P \implies P \vee Q$	Addition—add

3. [**3 points**] Using the given derivation rules, give a proof sequence to show the following wff is a tautology.

$$A' \wedge (A \vee B) \to B$$

1.	$A \vee B$	hyp
2.	$(A')' \vee B$	1, dn
3.	$A' \to B$	2, imp
4.	A'	hyp
5.	B	3,4 mp