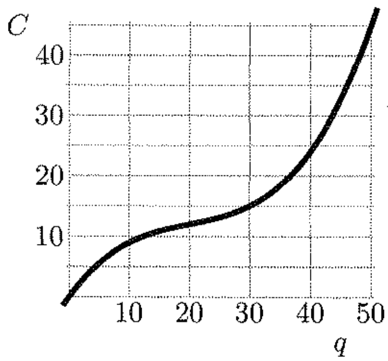
Name: Key

1. A graph of the total cost function $C(q)$ (in thousands of dollars) appears below.



(a) [**3 points**] Estimate the production level that minimizes marginal cost.

$q = 20$

(b) [**3 points**] Estimate the production level that minimizes average cost.

$q = 30$

2. [**7 points**] The cost of producing q units is given by $C(q) = 9q^3 - 225q^2 + 6875q$. Find the production level that minimizes average cost exactly.

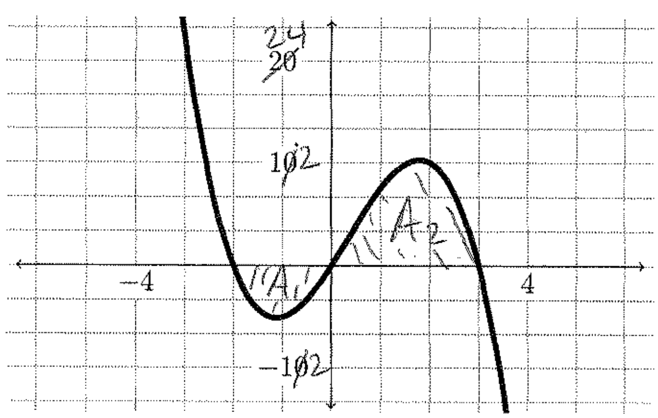
4 MC = AC

$27q^2 - 450q + \cancel{6875} = 9q^2 - 225q + \cancel{6875}$

$18q^{\cancel{2}} = 225\cancel{q}$

$q = \frac{225}{18} = \cancel{4B}\ 12.5$ units

3. [**7 points**] Use the graph of $f(t)$ to estimate the value of the integral $\int_{-2}^{3} f(t)\,dt$.



Each box ≈ 4

$A_1 \approx 2$ boxes

$A_2 \approx 6$ boxes

Total area ≈ $4 \cdot 4 = \boxed{16}$

4. **[8 parts, 3 points each]** Evaluate the following indefinite integrals.

(a) $\int 6\,dx$

$6x + C$

(b) $\int z - 3z^2\,dz$

$\frac{z^2}{2} - z^3 + C$

(c) $\int 2x^6(3x+1)\,dx$

$= \int 6x^7 + 2x^6\,dx$

$= \boxed{\frac{6}{8}x^8 + \frac{2}{7}x^7 + C}$

(d) $\int e^{7t}\,dt$

$= \boxed{\frac{1}{7}e^{7t} + C}$

(e) $\int \frac{1}{x^8}\,dx$

$= \int x^{-8}\,dx$

$= \boxed{-\frac{1}{7}x^{-7} + C}$

(f) $\int r^{11+\sqrt{2}}\,dr$

$= \boxed{\frac{r^{12+\sqrt{2}}}{12+\sqrt{2}} + C}$

(g) $\int e^3 x\,dx$

$= \boxed{e^3\frac{x^2}{2} + C}$

(h) $\int x^{-1}\,dx$

$= \boxed{\ln|x| + C}$

5. [**2 parts, 5 points each**] Evaluate the following indefinite integrals.

(a) $\int \frac{4x^3+3}{(x^4+3x+8)^5}\,dx = \int \frac{1}{\omega^5}\,d\omega$

$\omega = x^4+3x+8$

$\frac{d\omega}{dx} = 4x^3+3$

$= \int \omega^{-5}\,d\omega$

$= -\frac{1}{4}\omega^{-4}+C$

$= -\frac{1}{4\omega^4}+C$

$= \boxed{-\frac{1}{4(x^4+3x+8)^4}+C}$

(b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^{\omega}}{\cancel{\sqrt{x}}}\cdot 2\cancel{\sqrt{x}}\,d\omega$

$\omega = \sqrt{x}$

$= x^{1/2}$

$\frac{d\omega}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\,d\omega = dx$

$= 2\int e^{\omega}\,d\omega$

$= 2e^{\omega}+C$

$= \boxed{2e^{\sqrt{x}}+C}$

6. [**6 points**] Find the average value of the function $f(x) = x(4-x)$ over the interval $[0,4]$ exactly.

$$\text{Avg val} = \frac{1}{4-0}\int_0^4 x(4-x)\,dx$$

$$= \frac{1}{4}\int_0^4 4x - x^2\,dx$$

$$= \frac{1}{4}\left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4$$

$$= \frac{1}{4}\left[\left(2\cdot 4^2 - \frac{4^3}{3}\right) - 0\right]$$

$$= \frac{1}{4}\left[32 - \frac{64}{3}\right] = 8 - \frac{16}{3} = \boxed{\frac{8}{3}}$$

7. [**4 parts, 5 points each**] Use the Fundamental Theorem of Calculus to solve the following definite integrals exactly.

(a) $\int_{-2}^{1} 3x^2\,dx$

$$= x^3\Big|_{-2}^{1}$$

$$= 1^3 - (-2)^3$$

$$= 1 - (-8)$$

$$= 1 + 8 = \boxed{9}$$

(b) $\int_{2}^{4} t^3 - e^{2t}\,dt$

$$\left(\frac{t^4}{4} - \frac{1}{2}e^{2t}\right)\Big|_2^4$$

$$= \left(\frac{4^4}{4} - \frac{1}{2}e^8\right) - \left(\frac{2^4}{4} - \frac{1}{2}e^4\right)$$

$$= \left(64 - \tfrac{1}{2}e^8\right) - \left(4 - \tfrac{1}{2}e^4\right)$$

$$= \boxed{60 - \tfrac{1}{2}e^8 + \tfrac{1}{2}e^4}$$

(c) $\int_{2}^{5} \frac{(\ln x)^2}{x}\,dx$

$$= \int \frac{w^2}{\cancel{x}}\,\cancel{x}\,dw$$

$w = \ln x$

$\frac{dw}{dx} = \frac{1}{x}$

$x\,dw = dx$

$$= \frac{w^3}{3} + C$$

$$= \frac{(\ln x)^3}{3} + C$$

$$\int_2^5 \frac{(\ln x)^2}{2}\,dx = \frac{(\ln x)^3}{3}\Big|_2^5$$

$$= \boxed{\frac{(\ln 5)^3}{3} - \frac{(\ln 2)^3}{3}}$$

(d) $\int_0^1 (x + e^{2x})(x^2 + e^{2x})^{10}\,dx$

$w = x^2 + e^{2x}$

$\frac{dw}{dx} = 2x + 2e^{2x}$

$\frac{dw}{2x + 2e^{2x}} = dx$

$$= \int_{\cancel{0}}^{1} \cancel{(x+e^{2x})}\, w^{10}\, \frac{dw}{\cancel{2x+2e^{2x}}}$$

$$= \int \tfrac{1}{2} w^{10}\,dw$$

$$= \frac{1}{2}\frac{w^{11}}{11} + C$$

$$= \frac{1}{22}\left(x^2 + e^{2x}\right)^{11}\Big|_0^1$$

$$= \frac{1}{22}\left[(1^2 + e^2)^{11} - (0^2 + e^0)^{11}\right]$$

$$= \frac{1}{22}\left[(1 + e^2)^{11} - 1\right]$$

$$= \boxed{\frac{(1+e^2)^{11} - 1}{22}}$$

8. [**4 parts, 3 points each**] At time $t = 0$ hours, the surface of a pond begins to freeze. The rate R (in inches per hour) of growth in ice is a function $R(t)$ of time.

t	0	1	2	3	4	5	6	7	8
$R(t)$	0	0.5	1	2	1.5	1	0.5	0.25	0.5

(a) Express the total change in the thickness of the ice during the first 8 hours as a definite integral.

$$\boxed{\int_0^8 R(t)\,dt}$$

(b) With $n = 4$, find the Left Hand Sum (LHS) approximation to the above integral.

$$LHS = 0\cdot 2 + 1\cdot 2 + 1.5\cdot 2 + 0.5\cdot 2$$
$$= 0 + 2 + 3 + 1 = \boxed{6 \text{ inches}}$$

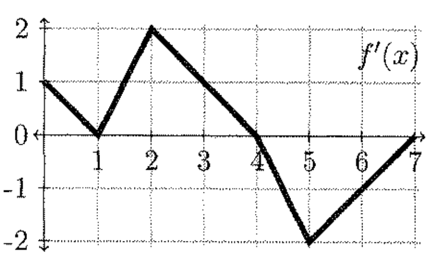
(c) With $n = 8$, find the Left Hand Sum (LHS) approximation to the above integral.

$$LHS = 0\cdot 1 + 0.5\cdot 1 + 1\cdot 1 + 2\cdot 1 + 1.5\cdot 1 + 1\cdot 1 + 0.5\cdot 1 + 0.25\cdot 1$$
$$= \tfrac{1}{2} + 1 + 2 + \tfrac{3}{2} + 1 + \tfrac{1}{2} + \tfrac{1}{4} = \boxed{6.75 \text{ in}}$$

(d) Which of these estimates would you expect to be more accurate? Briefly explain.

Expect the second to be more accurate since more ~~boxes~~ rectangles better captures the true area.

9. [**8 points**] The graph of the derivative $f'(x)$ is shown below. Fill in the table of values given that $f(0) = 4$.



x	0	1	2	3	4	5	6	7
$f(x)$	4	4.5	5.5	7	7.5	6.5	5	4.5