Name: Answer Key

1. [**10 parts, 2 points each**] Differentiate the following functions.

(a) $f(x) = 8$

$f'(x) = 0$

(b) $f(x) = 5x^2 - 2x + 1$

$f'(x) = 10x - 2$

(c) $f(x) = \frac{3}{x^2}$ $= 3x^{-2}$

$f'(x) = -6x^{-3}$

(d) $f(x) = 6^x$

$f'(x) = \ln(6) \cdot 6^x$

(e) $f(x) = e^{0.2x}$

$f'(x) = 0.2e^{0.2x}$

(f) $f(x) = 7\sqrt{x}$ $= 7x^{\frac{1}{2}}$

$f'(x) = \frac{7}{2} \cdot x^{-\frac{1}{2}}$

(g) $f(x) = 4\ln(x)$

$f'(x) = 4 \cdot \frac{1}{x}$

(h) $f(x) = e^{-x}$

$f'(x) = -e^{-x}$

(i) $f(x) = x^{\ln(4)}$

$f'(x) = \ln(4) \cdot x^{\ln(4)-1}$

(j) $f(x) = e^{\sqrt{5}-1}$ $\leftarrow$ Constant

$f'(x) = 0$

2. A pharmaceutical company finds that the cost C (in thousands of dollars) for producing q kilograms of a drug is given by the equation $C(q) = 2.2q^3 + 14.1q + 250$. The total revenue R (in thousands of dollars) received by the company when q kilograms are produced is given by the equation $R(q) = 8.2q^2 + 200q$.

   (a) [**4 points**] Find the marginal cost when the production q is 6 kilograms. Round your answer to 3 decimal places and include units.

   $$MC(q) = 6.6q^2 + 14.1$$
   $$MC(6) = 6.6 \cdot 36 + 14.1 = \boxed{251.700 \text{ thousand dollars per kg}}$$

   (b) [**4 points**] Find the marginal revenue when the production q is 6 kilograms. Round your answer to 3 decimal places and include units.

   $$MR(q) = 16.4q + 200$$
   $$MR(6) = 16.4 \cdot 6 + 200 = \boxed{298.400 \text{ thousand dollars per kg}}$$

   (c) [**2 points**] If 6 kilograms of the drug have already been produced, should the company produce more of the drug? Explain.

   Yes, since $MR(6) > MC(6)$, profits will increase if the company produces more of the drug.

3. Let $g(t) = \ln(1 + t^2)$.

   (a) [**5 points**] Find $g'(t)$.

   $$g'(t) = \frac{1}{1+t^2} \cdot \frac{d}{dt}\left[1+t^2\right]$$
   $$= \frac{1}{1+t^2} \cdot 2t = \boxed{\frac{2t}{1+t^2}}$$

   (b) [**5 points**] Find the equation of the tangent line to $g(t)$ at $t = 4$ exactly. Your answer may involve exponential functions, logarithmic functions, or both.

   Point: $(4, \ln(17))$

   Slope: $m = g'(4) = \frac{2 \cdot 4}{1+4^2} = \frac{8}{17}$

   $$y - y_0 = m(x - x_0)$$
   $$y - \ln(17) = \frac{8}{17}(x-4)$$
   $$\boxed{y = \frac{8}{17}(x-4) + \ln(17)}$$

4. [**4 parts, 5 points each**] Differentiate the following functions.

(a) $f(x) = \dfrac{x^3 - 8x + 2}{x^2 + 1}$

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}\left[x^3-8x+2\right] - (x^3-8x+2)\frac{d}{dx}\left[x^2+1\right]}{(x^2+1)^2}$$

$$= \boxed{\frac{(x^2+1)(3x^2-8) - (x^3-8x+2)(2x)}{(x^2+1)^2}}$$

(b) $f(x) = 2^{(x^3+4x)}$

$$f'(x) = \ln(2) \cdot 2^{(x^3+4x)} \cdot \frac{d}{dx}\left[x^3+4x\right]$$

$$= \boxed{\ln(2) \cdot 2^{(x^3+4x)} \cdot (3x^2+4)}$$

(c) $f(x) = (4x + \ln(x))^{-5}$

$$f'(x) = -5(4x+\ln(x))^{-6} \cdot \frac{d}{dx}\left[4x + \ln(x)\right]$$

$$= \boxed{-5(4x+\ln(x))^{-6} \cdot \left(4 + \frac{1}{x}\right)}$$

(d) $f(x) = \ln(\ln(x))$

$$f'(x) = \frac{1}{\ln(x)} \cdot \frac{d}{dx}\left[\ln(x)\right]$$

$$= \frac{1}{\ln(x)} \cdot \frac{1}{x} = \boxed{\frac{1}{x\ln(x)}}$$

5. [**5 parts, 4 points each**]

(a) Complete: a point p is a critical point of a function f if $f'(p) = 0$ or if $f'(p)$ is undefined.

(b) Let $f(x) = -2x^3 + 8x^2 - 8x$. Find $f'(x)$.

$$f'(x) = -6x^2 + 16x - 8$$
$$= -2(3x^2 - 8x + 4)$$
$$= -2(3x - 2)(x - 2)$$

(c) Find the critical points of f.

$$-2(3x-2)(x-2) = 0$$

$3x - 2 = 0$ or $x - 2 = 0$

$3x = 2$ $\qquad$ $x = 2$

$x = \frac{2}{3}$

$\boxed{\text{Critical points: } \tfrac{2}{3}, 2}$

(d) Find $f''(x)$.

$$f''(x) = \frac{d}{dx}\left[-6x^2 + 16x - 8\right]$$
$$= \boxed{-12x + 16}$$

(e) Using either the First Derivative Test or the Second Derivative Test, classify each critical point as a local minimum, a local maximum, or neither.

$\frac{2}{3}$: $f''\left(\frac{2}{3}\right) = -12 \cdot \frac{2}{3} + 16 = -8 + 16 = 8 > 0$

++ $\Rightarrow$ $\boxed{\text{So } f \text{ has a local min at } x = \tfrac{2}{3}.}$

2: $f''(2) = -12 \cdot 2 + 16 = -24 + 16 = -8 < 0$

$\Rightarrow$ $\boxed{\text{So } f \text{ has a local max at } x = 2.}$

6. Let $f(x) = (x-2)^3 e^x$.

(a) [**7 points**] Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[(x-2)^3\right]\cdot e^x + (x-2)^3 \frac{d}{dx}\left[e^x\right]$$

$$= 3(x-2)^2 \cdot \frac{d}{dx}\left[x-2\right]\cdot e^x + (x-2)^3 \cdot e^x$$

$$= 3(x-2)^2 \cdot 1 \cdot e^x + (x-2)^3 \cdot e^x$$

$$= e^x (x-2)^2 \left(3 + (x-2)\right) = \boxed{e^x (x-2)^2 (x+1)}$$

(b) [**7 points**] Find the critical points of f.

$$e^x (x-2)^2 (x+1) = 0$$

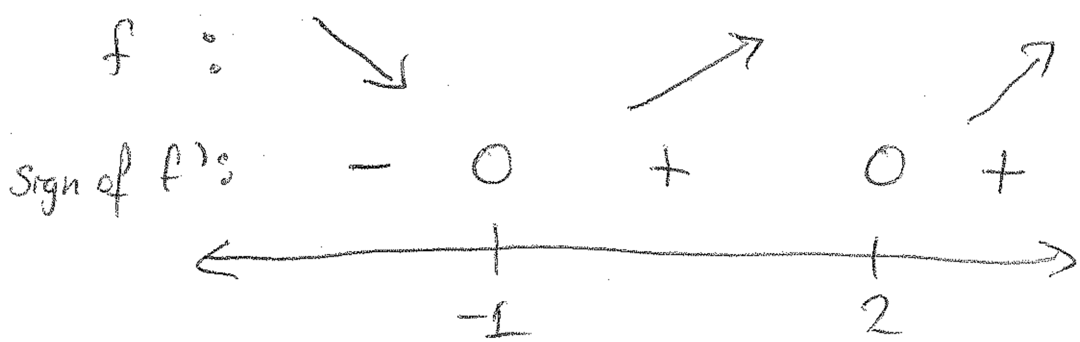
$e^x = 0$ or $(x-2)^2 = 0$ or $x+1 = 0$

No soln $\qquad x = 2 \qquad x = -1$

$\boxed{\text{So } f \text{ has critical points at } x=-1 \text{ and } x=2.}$

(c) [**6 points**] Use the First Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.

Sign Chart:



$\boxed{\begin{array}{l}\text{So } f \text{ has a local min at } x=-1 \\ \text{and } f \text{ has neither a local min nor a local max at } x=2.\end{array}}$

Name: ____________________

Do not turn the page until instructed.

Directions:

1. Write your name on this page and, after the test begins, on the first page of the test.

2. Round all numerical answers to three (3) decimal places.

3. Show your work unless you are instructed otherwise. No credit for answers without work.

4. You may use a calculator provided it is not equipped with a Computer Algebra System (CAS).

5. Turn off and put away all other electronic equipment (especially cell phones), notes, and books.

6. Good luck!