Name: Key

1. A manufacturer sells quality pens at a price of \$14 per pen, and at this price consumers have been buying 12,000 pens a month. The manufacturer estimates that for each \$1 increase in the price, 1,000 fewer pens will be sold. Similarly, for each \$1 decrease in price, 1,000 more pens will be sold. The manufacturer can produce the pens at a cost of ~~\$12~~ \$10 per pen.

   (a) **[3 points]** Express the monthly profit P as a function of the sale price x.

$$P(x) = x\cdot(\#\text{ of pens sold}) - 10\cdot(\#\text{ pens sold})$$
$$= (x-10)\cdot(\#\text{ pens sold}).$$

- Let y be the # pens sold at price x.
- Know: y is a linear function with slope -1000.
- Know: $(14, 12000)$ is a point on the line.

$$\Longrightarrow \quad y - y_1 = m(x - x_1)$$
$$y - 12000 = -1000(x-14)$$
$$y = -1000x + 14000 + 12000$$
$$= -1000x + 26000$$

- $\boxed{P(x) = (x-10)(-1000x+26000) = -1000x^2 + 36000x - 260{,}000.}$

   (b) **[1 point]** Find the sale price that maximizes the manufacturer's profit.

- $P(x)$ is a parabola that opens down, so $P(x)$ is maximized when

$$x = \frac{-b}{2a} = \frac{-36\cancel{000}}{-2\cancel{000}}$$
$$= 18$$

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- So sale price that maximizes profit is $\boxed{\$18}$.

2. **[6 parts, 1 point each]** Find the indicated limit if it exists. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$. Show your work.

(a) $\lim_{x\to -1/3} (2-3x^2) =$

(plug in)

$$= 2-3\left(-\tfrac{1}{3}\right)^2$$
$$= 2-3\cdot\tfrac{1}{9}$$
$$= 2-\tfrac{1}{3}$$
$$= \boxed{5/3}$$

(b) $\lim_{x\to -3} \dfrac{x^2+x-6}{x+3} =$

$$\lim_{x\to -3} \frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}}$$
$$= \lim_{x\to -3} x-2$$
$$= \boxed{-5}$$

(c) $\lim_{x\to -2} \dfrac{x^2+3x+2}{x^2-x-6} =$

$$= \lim_{x\to -2} \frac{\cancel{(x+2)}(x+1)}{(x-3)\cancel{(x+2)}}$$
$$= \frac{-2+1}{-2-3}$$
$$= \boxed{\tfrac{1}{5}}$$

(d) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} =$

$$\lim_{x\to 9} \frac{(\sqrt{x}-3)}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}$$
$$= \lim_{x\to 9} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$$
$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\tfrac{1}{6}}$$

(e) $\lim_{x\to 14} \dfrac{\sqrt{x+2}-4}{x-14} =$

$$\lim_{x\to 14} \frac{\sqrt{x+2}-4}{x-14}\cdot\frac{\sqrt{x+2}+4}{\sqrt{x+2}+4}$$
$$= \lim_{x\to 14} \frac{(x+2)-16}{(x-14)(\sqrt{x+2}+4)}$$
$$= \lim_{x\to 14} \frac{\cancel{x-14}}{\cancel{(x-14)}(\sqrt{x+2}+4)} = \frac{1}{\sqrt{14+2}+4} = \boxed{\tfrac{1}{8}}$$

(f) $\lim_{x\to -\infty} \dfrac{5x^2+30x+18}{4x+17} =$

$$= \lim_{x\to -\infty} \frac{5x^2+30x+18}{4x+17}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$
$$= \lim_{x\to -\infty} \frac{5x+30+18/x}{4+17/x} \quad \begin{array}{l}\to -\infty \\ \to 4\end{array}$$
$$= \boxed{-\infty}$$