Name: Key

1. **[2 parts, 2 points each]** Use integration by parts to find the given integral.

(a) $\int_0^1 \frac{x}{e^{3x}}\,dx$

$\int x e^{-3x}\,dx$

$u = x$ $\quad v = -\frac{1}{3}e^{-3x}$

$du = dx$ $\quad dv = e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} - \int -\frac{1}{3}e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\left(-\frac{1}{3}e^{-3x}\right)$

$= -\frac{1}{3}e^{-3x}\left(x + \frac{1}{3}\right) + C$

$\int_0^1 \frac{x}{e^{3x}}\,dx = \left(-\frac{1}{3}e^{-3x}\left(x+\frac{1}{3}\right)\right)\Big|_0^1$

$= \left(-\frac{1}{3}e^{-3}\left(\frac{4}{3}\right)\right) - \left(-\frac{1}{3}e^{0}\left(\frac{1}{3}\right)\right)$

$= -\frac{4}{9}\cdot\frac{1}{e^3} + \frac{1}{9}$

$= \frac{1}{9} - \frac{4}{9e^3} = \boxed{\frac{e^3-4}{9e^3}}$

(b) $\int x^2 \ln x\,dx$

$u = \ln x$ $\quad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx$ $\quad dv = x^2\,dx$

$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}dx$

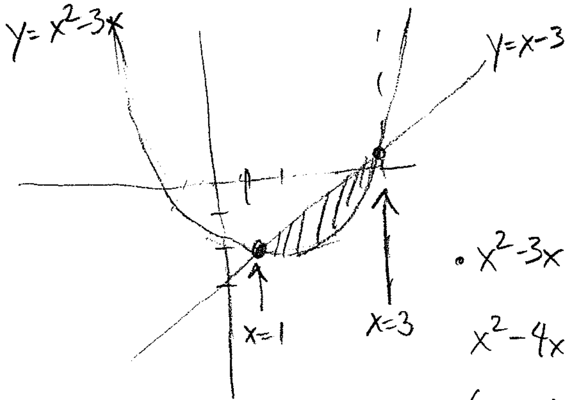
$= \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{3}\frac{x^3}{3}$

$= \boxed{\frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right) + C}$

OVER →

2. **[2 points]** Find the area of the region bounded by the curves $y = x^2 - 3x$ and $y = x - 3$.



$$x^2-3x = x-3$$
$$x^2-4x+3=0$$
$$(x-1)(x-3)=0$$
$$x=1,\ 3$$

$\underline{y=x^2-3x}$: parabola

$$x_{vert} = \frac{-b}{2a} = \frac{3}{2}$$

$$y_{vert} = \left(\tfrac{3}{2}\right)^2 - 3\cdot\tfrac{3}{2} = \tfrac{9}{4} - \tfrac{9}{2} = 9\left(\tfrac{1}{4}-\tfrac{1}{2}\right) = -\tfrac{9}{4}$$

$$A = \int_1^3 (x-3) - (x^2-3x)\,dx$$
$$= \int_1^3 x-3-x^2+3x\,dx$$
$$= \int_1^3 -x^2+4x-3\,dx$$
$$= \left(-\frac{x^3}{3} + 2x^2 - 3x\right)\Big|_1^3$$
$$= (-9+18-9) - \left(-\tfrac{1}{3}+2-3\right)$$
$$= 0 - (-4/3) = \boxed{4/3}$$

3. **[2 points]** Find the average value of the function $f(x) = \cancel{(2x-5)^7}$ $(4x-5)^6$ over the range $\cancel{2}$ 4 $\le x \le$ $\cancel{3}$ 6.

$1 \le x \le 3/2$

$$\text{Avg val} = \frac{1}{b-a}\int_a^b f(x)\,dx$$
$$= \frac{1}{3/2-1}\int_1^{3/2} (4x-5)^6\,dx$$
$$= \frac{1}{\frac{1}{2}}\left(\frac{1}{7}\cdot\frac{1}{4}(4x-5)^7\right)\Big|_1^{3/2}$$
$$= 2\left(\frac{1}{28}(4x-5)^7\right)\Big|_1^{3/2}$$
$$= 2\left[\frac{1}{28}(1)^7 - \frac{1}{28}(-1)^7\right]$$
$$= 2\left[\frac{1}{28}+\frac{1}{28}\right] = 2\left[\frac{2}{28}\right]$$
$$= \frac{4}{28} = \boxed{\frac{1}{7}}$$

OVER →

4. **[2 points]** Find the following integral using techniques of your choice.

$$\int \frac{1}{x \ln x} + e^{5x}\, dx$$

$$= \int \frac{1}{x\ln x}\,dx + \int e^{5x}\,dx$$

$$= \int \frac{1}{x\ln x}\,dx + \frac{1}{5}e^{5x}$$

Attack with a substitution:

- $u = \ln x$
- $du = \frac{1}{x}dx$
- $dx = x\,du$

$$= \int \frac{1}{\cancel{x}\cdot u}\cdot \cancel{x}\,du + \frac{1}{5}e^{5x}$$

$$= \int \frac{1}{u}\,du + \frac{1}{5}e^{5x}$$

$$= \ln|u| + \frac{1}{5}e^{5x}$$

$$= \boxed{\ln|\ln x| + \frac{1}{5}e^{5x} + C}$$