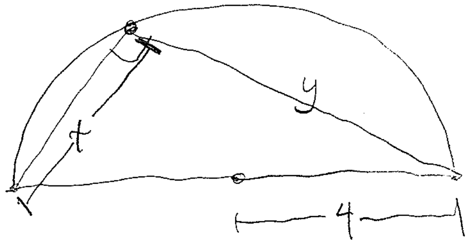
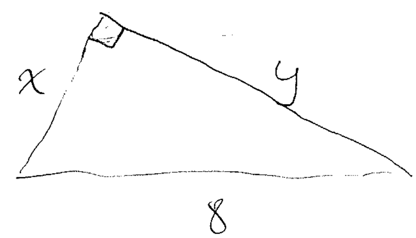
3.5 #14



- Let x be the length of one of the sides of the triangle, let y be the other.
- We have a right triangle; one leg has length x and hypotenuse has length 8:



- Pythagorean Theorem:

$$x^2 + y^2 = 8^2$$

- Area of triangle:

$$A = \frac{xy}{2}$$

Implicit differentiation:

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

- Want to maximize A as x ranges from 0 to 8.

(1) Find critical points.

- $$\frac{dA}{dx} = \frac{1}{2}\left[x \cdot \frac{dy}{dx} + (1)\cdot y\right]$$

$$= \frac{1}{2}\left[-\frac{x^2}{y} + y\right]$$

- $\frac{dA}{dx}$ exists except when $y=0$

- Set $\frac{dA}{dx} = 0$:

$$\frac{1}{2}\left[-\frac{x^2}{y} + y\right] = 0$$

$$y = \frac{x^2}{y}$$

$$y^2 = x^2$$

$$y^2 - x^2 = 0$$

$$(y+x)(y-x) = 0$$

$y+x=0$ or $y-x=0$

No soln: x, y are both pos. $\quad x=y$.

(2) Evaluate A when $x=0$, $x=y$, and $x=8$.

When $x=y$:

$$x^2+y^2=64$$

$$x^2+x^2=64$$

$$2x^2=64$$

$$x^2=32$$

$$x=\pm\sqrt{32}$$

But $x \geq 0$, so $x=\sqrt{32}=4\sqrt{2}$.

When $x=8$,

$$8^2+y^2=64$$

$$y^2=0$$

$$y=0.$$

So:

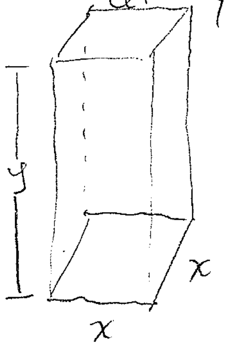
$\underline{x=0}$: $A = \frac{0 \cdot y}{2} = 0$

$\underline{x=y}$: $A = \frac{\sqrt{32}\cdot\sqrt{32}}{2} = \frac{32}{2} = 16$

$\underline{x=8}$: $A = \frac{8 \cdot 0}{2} = 0$

(3) So area of triangle is maximized when $\boxed{x=y=4\sqrt{2}}$.

\# 16. • Let x be the length of the sides in the square base;
Let y be the height.



• Volume $= x^2 y$

• $250 = x^2 y$

• $y = \dfrac{250}{x^2}$.

• Cost of box = cost of top, bottom + cost of sides

$$= 2(\text{Area of top, bottom}) + 1(\text{Area of sides})$$

$$= 2(2 \cdot x^2) + 1 \cdot (4 \cdot x \cdot y)$$

$$= 4x^2 + 4xy$$

$$= 4x^2 + 4\cancel{x} \cdot \frac{250}{x^{\cancel{2}}}$$

$$C(x) = 4\left(x^2 + \frac{250}{x}\right)$$

• Minimize cost of box ~~for~~ x in range $(0, \infty)$.

• $C'(x) = 4(2x - 250x^{-2}) = 4\left(2x - \dfrac{250}{x^2}\right) = 8\left(x - \dfrac{125}{x^2}\right)$

• Compute sign chart for C': Note: C' continuous except when $x=0$.

$$C'(x) = 0$$

$$8\left(x - \frac{125}{x^2}\right) = 0$$

$$x - \frac{125}{x^2} = 0$$

$$x^2\left(x - \frac{125}{x^2}\right) = 0 \cdot x^2$$

$$x^3 - 125 = 0$$

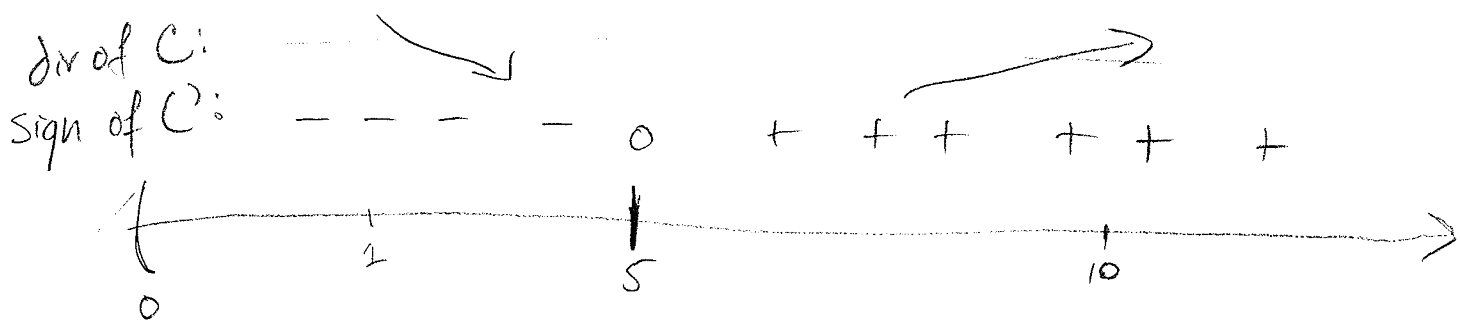
$$x^3 = 125$$

$$x = (125)^{1/3}$$

[Note: because 3 is odd, we only get one solution, not $x = \pm(125)^{1/3}$]

$$= 5$$

- $C'(1) = 8\left(1 - \frac{125}{1^2}\right) = 8(-124) < 0$
- $C'(10) = 8\left(10 - \frac{125}{100}\right) > 0$



- So: cost decreases on $(0, 5)$ and increases on $(5, \infty)$.

Therefore: cost is minimized when $x = 5$.

- Minimum cost:

$$C(5) = 4\left(5^2 + \frac{250}{5}\right) = 4(25 + 50) = 4 \cdot 25(1+2)$$
$$= 100 \cdot 3$$
$$= 300.$$

Answer: $\boxed{\text{No,}}$ the minimum cost for the box is exactly \$300.