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Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[9 parts, 1 point each]** Decide whether the following are true or false. Indicate your answer by writing the entire word. No justification required.

$A = \{\}$ $B = \{2\}$ $C = \{\varnothing, \varnothing\}$ $D = \{\{\varnothing\}, 2\}$ $E = \{\varnothing, \{2\}\}$

(a) $2 \in B$	(d) $C \in D$	(g) $A \subseteq B$
(b) $A \in C$	(e) $B \in D$	(h) $C \subseteq A$
(c) $A \in D$	(f) $2 \subseteq B$	(i) $C \subseteq D$

2. **[3 parts, 7 points each]** Give full definitions, using appropriate notation, for the following.

   (a) The Cartesian product of sets $A_1, \ldots, A_n$

   (b) The intersection of sets A and B

   (c) The power set of A

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3. **[2 parts, 3 points each]** Sketch the following sets in the plane.

(a) $(-1, 4) \times [-2, 1]$

(b) $\{(x, y) \in \mathbb{R}^2 : \ x \in \mathbb{Z} \text{ or } y < 1\}$

4. **[6 parts, 3 points each]** Express each set by listing the elements between braces.

$A = \{\{\}, 2, \{1\}, \{2, 2\}\}$ $B = \{\{1, 1\}, \{2, 3\}, (2, 3), 2\}$

$C = \{\{2\}, \{3, 2\}, \varnothing\}$ $D = \{\varnothing, \{1, 2\}, \{2, 3\}, (3, 2)\}$

(a) $A \cap B$

(b) $B \cap C$

(c) $(B \cup C) - A$

(d) $(C - A) \times C$

(e) $\mathcal{P}(C \cap D)$

(f) $B \cap \mathcal{P}(\mathbb{Z})$

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5. **[4 parts, 3 points each]** Give Venn Diagrams for each of the following sets relative to a universe U.

   (a) $A \cap (B \cup C)$

   (b) $(A \cup B \cup C) - (B \cup C)$

   (c) $A - (B - C)$

   (d) $\overline{A \cup B} \cap \overline{B - C}$

6. **[5 points]** Give two examples of an infinite set A such that $A \in \mathcal{P}(\mathcal{P}(\mathbb{Z}))$.

7. **[5 points]** Use Venn Diagrams to decide if the equation $(A - B) - C = A \cap \overline{B} \cap \overline{C}$ is valid for all sets A, B, and C.

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8. **[3 parts, 6 points each]** Let $I = \{\alpha \in \mathbb{R}\colon\ \alpha > 0\}$ and let $D_\alpha = \{(x, y) \in \mathbb{R}^2\colon\ y \geq \alpha x\}$.

   (a) Sketch the example sets $D_{1/2}$, D_1, and D_2.

   (b) Sketch $\bigcap_{\alpha \in I} D_\alpha$.

   (c) Sketch $\bigcup_{\alpha \in I} D_\alpha$.

9. **[6 parts, 1 point each]** Determine whether the following statements and are true or false; write the entire word. Give a brief justification.

   $P(X)$: The set X is finite $\qquad$ $Q : 1 + 1 = 3$ $\qquad$ $R(a) : a$ is an even integer

   (a) Q

   (b) $P(\varnothing)$

   (c) $R(3) \vee Q$

   (d) $(\sim Q) \wedge R(4)$

   (e) $P(\{a \in \mathbb{Z}\colon\ R(a)\})$

   (f) $P(\{X \subseteq \mathbb{Z}\colon\ |X| \leq 5\})$