Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[9 parts, 1 point each]** Decide whether the following are true or false. Indicate your answer by writing the entire word. No justification required.

$A = \{\}$ $B = \{2\}$ $C = \{\varnothing, \varnothing\}$ $D = \{\{\varnothing\}, 2\}$ $E = \{\varnothing, \{2\}\}$

(a) $2 \in B$

TRUE

(b) $A \in C$

TRUE $A = \varnothing \in C$

(c) $A \in D$

FALSE $A = \varnothing$, $\varnothing \neq \{\varnothing\}$

(d) $C \in D$

TRUE $C = \{\varnothing\} \in D$

(e) $B \in D$

FALSE $B = \{2\} \notin D$ since $\{2\} \neq 2$

(f) $2 \subseteq B$

FALSE 2 is not a set.

(g) $A \subseteq B$

TRUE $\varnothing$ is a subset of every set

(h) $C \subseteq A$

FALSE $|C| = 1$ but $|A| = 0$

(i) $C \subseteq D$

FALSE $\varnothing \in C$ but $\varnothing \notin D$.

2. **[3 parts, 7 points each]** Give full definitions, using appropriate notation, for the following.

(a) The Cartesian product of sets $A_1, \ldots, A_n$

$A_1 \times \cdots \times A_n = \{(a_1, \ldots, a_n) : a_i \in A_i \text{ for each } i \in \{1, \ldots, n\}\}$

(b) The intersection of sets A and B

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

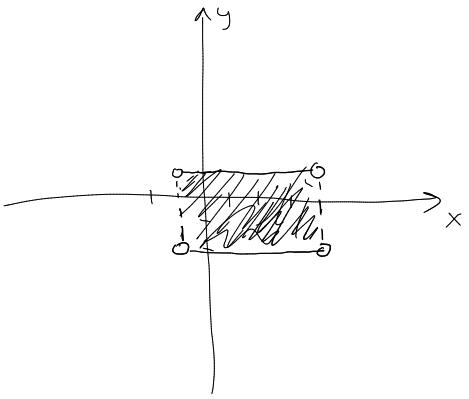
(c) The power set of A

$\mathcal{P}(A) = \{X : X \subseteq A\}$

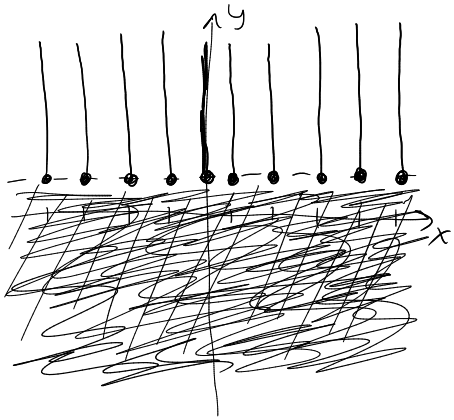
Name: _____

3. **[2 parts, 3 points each]** Sketch the following sets in the plane.

(a) $(-1,4)\times[-2,1] = \{(x,y): -1<x<4 \text{ and } -2\le y\le 1\}$



(b) $\{(x,y)\in\mathbb{R}^2: x\in\mathbb{Z} \text{ or } y<1\}$



4. **[6 parts, 3 points each]** Express each set by listing the elements between braces.

$A = \{\{\}, 2, \{1\}, \{2,2\}\}$ $\quad$ $B = \{\{1,1\}, \{2,3\}, (2,3), 2\}$

$C = \{\{2\}, \{3,2\}, \varnothing\}$ $\quad$ $D = \{\varnothing, \{1,2\}, \{2,3\}, (3,2)\}$

(a) $A\cap B$

$= \boxed{\{2, \{1\}\}}$

Note: $\{1,1\} = \{1\}$

(b) $B\cap C$

$= \boxed{\{\{2,3\}\}}$

(c) $(B\cup C) - A$

$B\cup C = \{\{1\}, \{2,3\}, (2,3), 2, \{2\}, \varnothing\}$

$A = \{\{1\}, 2, \{2\}, \varnothing\}$

$(B\cup C) - A = \boxed{\{\{2,3\}, (2,3)\}}$

(d) $(C-A)\times C$

$C - A = \{\{2\}, \{2,3\}, \varnothing\} - \{\varnothing, 2, \{1\}, \{2\}\} = \{\{2,3\}\}$

$(C-A)\times C = \{\{2,3\}\} \times \{\{2\}, \{2,3\}, \varnothing\} =$

$\boxed{\{(\{2,3\}, \{2\}), (\{2,3\}, \{2,3\}), (\{2,3\}, \varnothing)\}}$

(e) $\mathcal{P}(C\cap D)$

$C\cap D = \{\overbrace{\{2,3\}}^{a}, \overbrace{\varnothing}^{b}\}$

$\mathcal{P}(C\cap D) = \{\varnothing, \{a\}, \{b\}, \{a,b\}\}$

$= \boxed{\{\varnothing, \{\{2,3\}\}, \{\varnothing\}, \{\{2,3\}, \varnothing\}\}}$

(f) $B\cap\mathcal{P}(\mathbb{Z})$

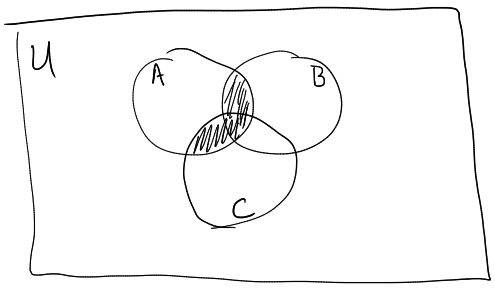
$B\cap\mathcal{P}(\mathbb{Z})$ is the set of elements in B that are subsets of $\mathbb{Z}$, so

$B\cap\mathcal{P}(\mathbb{Z}) = \boxed{\{\{1\}, \{2,3\}\}}$

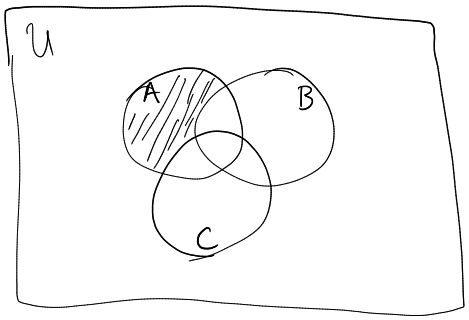
Name: _____

5. **[4 parts, 3 points each]** Give Venn Diagrams for each of the following sets relative to a universe U.

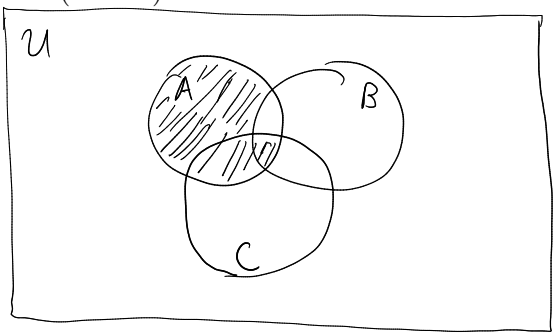
(a) $A \cap (B \cup C)$



(b) $(A \cup B \cup C) - (B \cup C)$



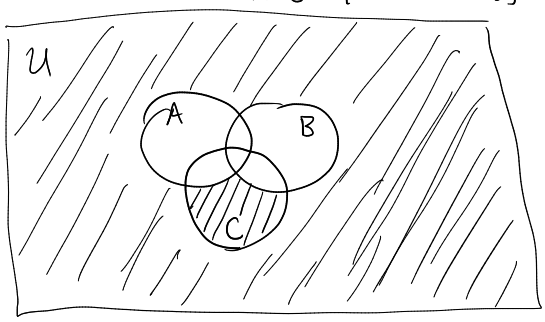
(c) $A - (B - C)$



(d) $\overline{A \cup B} \cap \overline{B - C}$

$\overline{B-C} = \overline{\{x : x \in B \text{ and } x \notin C\}} = \{x : x \notin B \text{ or } x \in C\}$

$\overline{A \cup B} = \overline{\{x : x \in A \text{ or } x \in B\}} = \{x : x \notin A \text{ and } x \notin B\}$



6. **[5 points]** Give two examples of an infinite set A such that $A \in \mathcal{P}(\mathcal{P}(\mathbb{Z}))$.

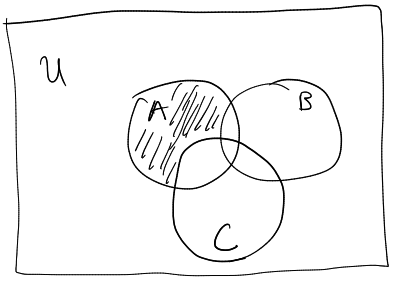
$A \in \mathcal{P}(\mathcal{P}(\mathbb{Z})) \Leftrightarrow A \in \mathcal{P}(\mathbb{Z}) \Leftrightarrow$ Every elt in A is in $\mathcal{P}(\mathbb{Z}) \Leftrightarrow$ Every elt in A is a subset of $\mathbb{Z}$.

Ex 1: $A = \{X \subseteq \mathbb{Z} : |X| = 1\} = \{\ldots, \{-2\}, \{-1\}, \{0\}, \{1\}, \{2\}, \ldots\}$

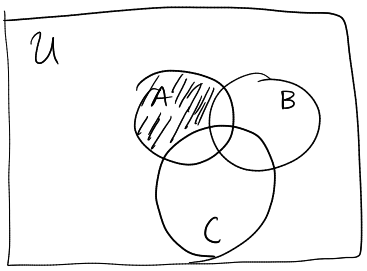
Ex 2: $A = \{X : X \subseteq \mathbb{Z}\} = \mathcal{P}(\mathbb{Z})$

7. **[5 points]** Use Venn Diagrams to decide if the equation $(A - B) - C = A \cap \overline{B} \cap \overline{C}$ is valid for all sets A, B, and C.

$(A-B)-C$:



$A \cap \overline{B} \cap \overline{C}$:

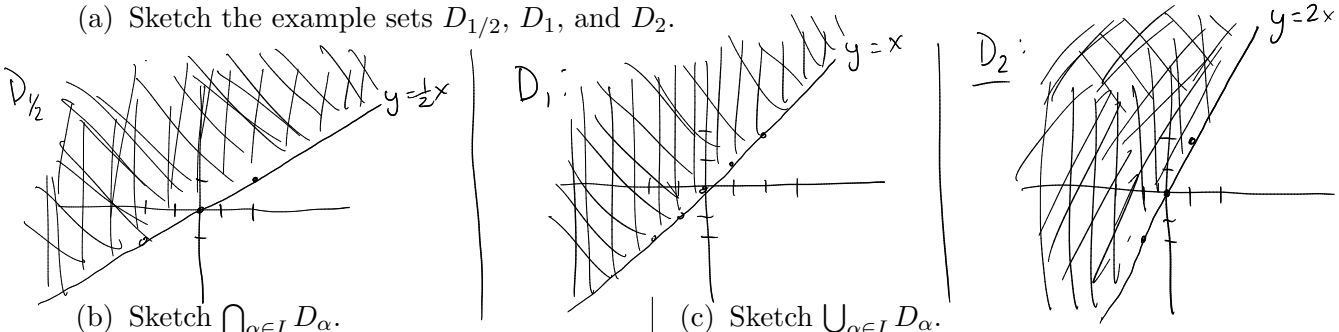


Yes, $(A-B)-C = A \cap \overline{B} \cap \overline{C}$ for all sets A, B, C.

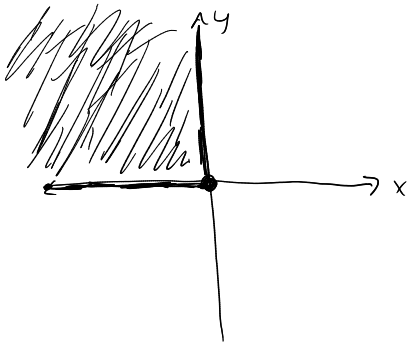
Name: _____

8. **[3 parts, 6 points each]** Let $I = \{\alpha \in \mathbb{R}\colon\ \alpha > 0\}$ and let $D_\alpha = \{(x, y) \in \mathbb{R}^2\colon\ y \geq \alpha x\}$.

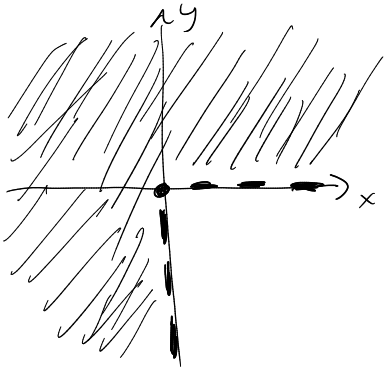
(a) Sketch the example sets $D_{1/2}$, D_1, and D_2.



(b) Sketch $\bigcap_{\alpha\in I} D_\alpha$.



(c) Sketch $\bigcup_{\alpha\in I} D_\alpha$.



9. **[6 parts, 1 point each]** Determine whether the following statements and are true or false; write the entire word. Give a brief justification.

$P(X)$: The set X is finite $\qquad$ $Q : 1 + 1 = 3$ $\qquad$ $R(a)$: a is an even integer

(a) Q

FALSE, $1+1=2$, not 3

(b) $P(\varnothing)$

TRUE, $\varnothing$ has size 0, so $\varnothing$ is a finite set

(c) $R(3) \vee Q$

FALSE R(3) and Q are both false.

(d) $(\sim Q) \wedge R(4)$

TRUE Q is false so $\sim Q$ is true. Also 4 is even. So both $\sim Q$ and $R(4)$ are true.

(e) $P(\{a \in \mathbb{Z}\colon\ R(a)\})$

$\{a \in \mathbb{Z} : R(a)\}$ is the set of even integers. This is an infinite set, so FALSE.

(f) $P(\{X \subseteq \mathbb{Z}\colon\ |X| \leq 5\})$

$\{X \subseteq \mathbb{Z} : |X| \leq 5\}$ is the set of all sets of at most 5 integers, so this is an infinite set. So FALSE.