Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 points]** Let $x, y \in \mathbb{Z}$. Give a proof by contrapositive that if xy is even, then x is even or y is even.

2. **[2 parts, 2 points each]** Congruences.

   (a) Give the definition of $a \equiv b \pmod{m}$.

   (b) Let $a, b, m \in \mathbb{Z}$ and let m be odd. Show that if $2a \equiv 2b \pmod{m}$, then $a \equiv b \pmod{m}$.

3. **[2 points]** Let $a \in \mathbb{Z}$. Prove that if $9 \nmid a^2$, then $3 \nmid a$.

4. **[2 points]** Prove that for each $n \in \mathbb{N}$, there exist nonnegative integers r and s such that s is odd and $n = 2^r s$.