

Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] We define the following statements and open sentences.

$$\begin{array}{ll} P: 0 = 1 & Q(x): x > 0 \\ R(a, b, c): a^2 + b^2 = c^2 & S(A): A \text{ is a finite set.} \end{array}$$

Decide whether the following are true or false; indicate your answer by writing the entire word “true” or the entire word “false”. Give brief justifications for partial credit.

(a) $\sim P$

(b) $P \implies Q(1)$

(c) $(P \vee R(3, 4, 5)) \implies Q(0)$

(d) $Q(0) \iff P$

(e) $Q((-1)^2) \implies S(\mathcal{P}(\{1, 2, 3, 4, 5\}))$

(f) $\sim S(\{x \in \mathbb{R}: x^2 - 6x + 2 = 0\}) \iff (R(3, 2, 1) \implies Q(2))$

2. [2 parts, 1 point each] Truth tables and logical equivalence.

(a) Write a truth table for $P \implies (Q \implies R)$

(b) Give a simple statement using only logical operands in $\{\sim, \vee, \wedge\}$ which is logically equivalent to $P \implies (Q \implies R)$.

3. [2 parts, 1 point each] Let P , Q , and R be statements. Use the standard logical operands $\sim, \vee, \wedge, \implies, \iff$ to express the following statements.

(a) If any one of the statements in $\{P, Q, R\}$ is true, then they all are true.

(b) Whenever two of the statements in $\{P, Q, R\}$ are true, the third is also true.