

Name: Solutions**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] We define the following statements and open sentences.

$$P: 0 = 1$$

$$Q(x): x > 0$$

$$R(a, b, c): a^2 + b^2 = c^2$$

$$S(A): A \text{ is a finite set.}$$

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". Give brief justifications for partial credit.

(a) $\sim P$ Since P is false, $\sim P$ is true.

$$P \Rightarrow Q:$$

T	T
T	F
F	T
F	F

✓
✓
✓

(b) $P \Rightarrow Q(1)$ Since P is false, $P \Rightarrow Q(1)$ is true.
 Also, since $Q(1)$ is true, $P \Rightarrow Q(1)$ is true.

(c) $(P \vee R(3, 4, 5)) \Rightarrow Q(0)$

↑ ↑ ↑
false true false

Since $R(3, 4, 5)$ is true, the hypothesis $P \vee R(3, 4, 5)$ is true. But the conclusion $Q(0)$ is false. So the implication is false.

(d) $Q(0) \Leftrightarrow P$

↑ ↑
false false

Since both sides of the biconditional are false, the statement is true.

(e) $Q((-1)^2) \Rightarrow S(P(\{1, 2, 3, 4, 5\}))$

Note: $Q((-1)^2)$ is $Q(1)$, or $1 > 0$, which is true.

$P(\{1, 2, 3, 4, 5\})$ is a finite set (it has 32 elements), so $S(P(\{1, 2, 3, 4, 5\}))$ is true.

The implication has the form true $\Rightarrow$ true and is therefore true.

(f) $\sim [S(\{x \in \mathbb{R}: x^2 - 6x + 2 = 0\})] \Leftrightarrow (R(3, 2, 1) \Rightarrow Q(2))$

Let $A = \{x \in \mathbb{R}: x^2 - 6x + 2 = 0\}$. Note that $|A| \leq 2$, so $S(A)$ is true and $\sim S(A)$ is false.

Also, $R(3, 2, 1)$ is false ($3^2 + 2^2 \neq 1^2$) and $Q(2)$ is true, so $R(3, 2, 1) \Rightarrow Q(2)$ is true. The biconditional has the form false $\Leftrightarrow$ true and is therefore false.

2. [2 parts, 1 point each] Truth tables and logical equivalence.

Note: $Q \Rightarrow R$ is equiv. to $\sim Q \vee R$

- (a) Write a truth table for $P \Rightarrow (Q \Rightarrow R)$

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

- (b) Give a simple statement using only logical operands in $\{\sim, \vee, \wedge\}$ which is logically equivalent to $P \Rightarrow (Q \Rightarrow R)$. Multiple answers possible, like:

$$\sim(P \wedge Q \wedge \sim R)$$

$$\sim P \vee \sim Q \vee R$$

3. [2 parts, 1 point each] Let P , Q , and R be statements. Use the standard logical operands $\sim, \vee, \wedge, \Rightarrow, \Leftrightarrow$ to express the following statements.

- (a) If any one of the statements in $\{P, Q, R\}$ is true, then they all are true.

$$P \vee Q \vee R \Rightarrow P \wedge Q \wedge R$$

Also: $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge (R \Rightarrow P)$

$$P \Leftrightarrow Q \Leftrightarrow R$$

- (b) Whenever two of the statements in $\{P, Q, R\}$ are true, the third is also true.

$$(P \wedge Q \Rightarrow R) \wedge (P \wedge R \Rightarrow Q) \wedge (Q \wedge R \Rightarrow P)$$

or

$$(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R) \Rightarrow P \wedge Q \wedge R$$