

Name: Solutions**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] Let the universe U be $\{-3, -2, -1, 0, 1, 2, 3\}$. Let $A = \{-1, 0, 1\}$, $B = \{1, 2\}$, and $C = \{-1, 2, 3\}$. Find the following sets.

(a) $B \cap A$

$$B \cap A = \{1, 2\} \cap \{-1, 0, 1\} = \{1\}$$

(b) $A - B$

$$A - B = \{-1, 0, 1\} - \{1, 2\} = \{-1, 0\}$$

(c) $B \cup \bar{A}$

$$B \cup \bar{A} = \{1, 2\} \cup \{-3, -2, 2, 3\} = \{-3, -2, 1, 2, 3\}$$

(d) $(B \times C) - (C \times B)$

$$B \times C = \{1, 2\} \times \{-1, 2, 3\} = \{(1, -1), (1, 2), (1, 3), (2, -1), (2, 2), (2, 3)\}$$

$$C \times B = \{-1, 2, 3\} \times \{1, 2\} = \{(-1, 1), (2, 1), (3, 1), (-1, 2), (2, 2), (3, 2)\}$$

$$(B \times C) - (C \times B) = \{(1, -1), (1, 2), (1, 3), (2, -1), (2, 3)\}$$

(e) $\mathcal{P}(A) - \mathcal{P}(B \cup C)$

$$A = \{-1, 0, 1\}, \quad B \cup C = \{-1, 1, 2, 3\}$$

$\mathcal{P}(A) - \mathcal{P}(B \cup C)$ is the set of all X such that $X \subseteq A$ but $X \not\subseteq B \cup C$. These are the subsets of A that contain 0: $\{\{0\}, \{0, -1\}, \{0, 1\}, \{0, -1, 1\}\}$

$$A = \{-1, 0, 1\} \quad B = \{1, 2\} \quad C = \{-1, 2, 3\}$$

(f) $\mathcal{P}(A) - (\mathcal{P}(B) \cup \mathcal{P}(C))$

$\mathcal{P}(A) - (\mathcal{P}(B) \cup \mathcal{P}(C))$ is the set of all X such that $X \subseteq A$, but $X \not\subseteq B$ and $X \not\subseteq C$.

So $X \subseteq A$ and X has an element in $A - B$, where $A - B = \{-1, 0, 1\} - \{1, 2\} = \{-1, 0\}$ and an elt in $A - C$, where $A - C = \{-1, 0, 1\} - \{-1, 2, 3\} = \{0, 1\}$.

$$\text{So } \{\{0\}, \{0, -1\}, \{0, 1\}, \{0, -1, 1\}, \{1, -1\}\}$$

2. [1 point] Is it true that $(A - B) \times (C - D)$ equals $(A \times C) - (B \times D)$ for all sets A, B, C, D ? Explain why or why not.

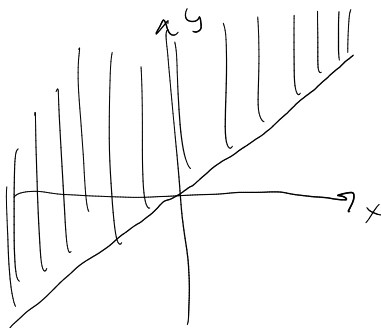
No, it is not true. For example, if $A = \{1, 2\}$, $B = \{2\}$, $C = \{3, 4\}$ and $D = \{4\}$, then

$$(A - B) \times (C - D) = \{1\} \times \{3\} = \{(1, 3)\}$$

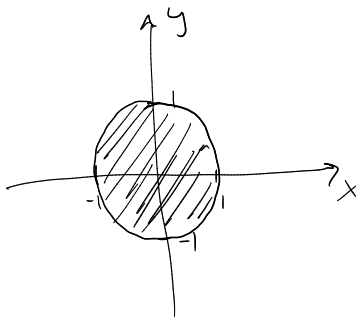
$$\text{but } (A \times C) - (B \times D) = (\{1, 2\} \times \{3, 4\}) - (\{2\} \times \{4\}) = \{(1, 3), (1, 4), (2, 3), (2, 4)\} - \{(2, 4)\} \\ = \{(1, 3), (1, 4), (2, 3)\}.$$

3. [3 parts, 1 point each] Let $A = \{(x, y) \in \mathbb{R}^2: y \geq x\}$ and $B = \{(x, y) \in \mathbb{R}^2: x^2 + y^2 \leq 1\}$. Sketch the following sets in the plane, using solid lines to represent boundaries that are in the set, and dashed lines/open circles to represent boundaries that are not in the set.

(a) A



(b) B



(c) $B - A$

