Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[3 points]** Recall that an integer p is *prime* if $p \geq 2$ and the only positive divisors of p are 1 and p. Let $a, b \in \mathbb{Z}$. Prove that $\gcd(a, b) = 1$ if and only if for each prime p, either $p \nmid a$ or $p \nmid b$. (You may use that each $n \in \mathbb{Z}$ with $n \geq 2$ is divisible by some prime.)

2. Existence and Uniqueness.

   (a) **[3 points]** Prove that there is a unique integer d such that $d > 0$ and d divides every integer.

   (b) **[1 point]** What goes wrong if the condition $d > 0$ is omitted from the statement to be proved in part (a)?

3. **[3 points]** Let A, B, C be sets. Let $X = A \cup (B \cap C)$ and let $Y = (A \cup B) \cap (A \cup C)$. Prove that $X = Y$.