

Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See “Guidelines and advice” on the course webpage for more information.

1. [BP 2.3] Express each of the following as a sentence of the form “If P , then Q ” without changing the meaning.
 - (a) For a function to be continuous, it is sufficient that it is differentiable.
 - (b) Whenever a surface has only one side, it is non-orientable.
 - (c) A geometric series with ratio r converges if $|r| < 1$.
 - (d) The discriminant is negative only if the quadratic equation has no real solutions.
 - (e) For a matrix to be invertible, it is necessary that no column has all zero entries.
2. [BP 2.4] Express each of the following as a sentence of the form “ P if and only if Q ” without changing the meaning.
 - (a) If a function has a constant derivative, then it is linear, and conversely.
 - (b) If $a \in \mathbb{Q}$ then $5a \in \mathbb{Q}$, and if $5a \in \mathbb{Q}$ then $a \in \mathbb{Q}$.
3. [BP 2.5]
 - (a) Write truth tables for the following statements.

i. $(Q \vee R) \iff (R \wedge Q)$

ii. $(P \wedge \sim P) \wedge Q$
 - (b) Suppose that the statement $((P \wedge Q) \vee R) \implies (R \vee S)$ is false. Determine the truth values of P , Q , R , and S . (Hint: try to solve without using a truth table.)
4. [BP 2.6] Decide whether or not the following pairs of statements are logically equivalent.
 - (a) $(P \implies Q) \vee R$ and $\sim((P \wedge \sim Q) \wedge \sim R)$
 - (b) $\sim(P \implies Q)$ and $P \wedge \sim Q$
 - (c) $P \wedge (Q \vee \sim Q)$ and $(\sim P) \implies (Q \wedge \sim Q)$
5. Suppose that x is a real number and $x = \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \dots}}}}$. Determine x .
6. You stand before a king who is completely honest and logical, but also a bit stingy. The king is in a good mood, and promises you the following: “If you tell me a statement which is true, then I will give you x gold coins, for some integer x such that $1 \leq x \leq 500$. If you tell me a false statement, then I will give you nothing. If you tell me a statement which leaves me with no logically consistent options, then I will chop off your head.” What should you say to the king to stay alive and get as many coins as possible?