

Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See “Guidelines and advice” on the course webpage for more information.

1. Let $A = \{1, 2, 3\}$ and $B = \{\sin, \cos\}$. List the elements of the following sets.

(a) $B \times A$	(c) $B \times A \times \emptyset$	(e) moved to HW3
(b) $B \times (A \times B)$	(d) $A \times \{\emptyset\}$	(f) moved to HW3

2. List the subsets of the following sets.

(a) $\{\mathbb{R}, \mathbb{N}, \mathbb{Q}\}$	(b) $\emptyset$	(c) $\{\{\mathbb{N}\}\}$
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3. Express the set $\{X \subseteq \mathbb{N}: |X| \leq 1\}$ by listing its elements between braces, using ellipses if necessary.

4. Decide whether the following statements are true or false. Give explanations.

(a) $\mathbb{R}^2 \subseteq \mathbb{R}^3$
(b) $\{(x, y) \in \mathbb{R}^2: x^2 - x = 0\} \subseteq \{(x, y) \in \mathbb{R}^2: x - 1 = 0\}$

5. Let $A = \{1, 2, 3\}$ and $B = \{\sin, \cos\}$. List the elements of the following sets.

(a) $\mathcal{P}(B)$	(b) $\mathcal{P}(B \times \{a\})$
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6. Suppose that $|A| = m$ and $|B| = n$. Find the given cardinalities.

(a) $ \mathcal{P}(\mathcal{P}(A)) $	(c) $ \mathcal{P}(A) \times \mathcal{P}(B) $
(b) $ \mathcal{P}(A \times \mathcal{P}(B)) $	(d) $ \{X \subseteq \mathcal{P}(A): X \leq 1\} $

7. You have two strings of fuse. When lit at one end, each will burn for exactly one hour. The fuses are not necessarily identical, though, and do not burn at a constant rate. All you have with you is a lighter and these two fuses. Can you measure exactly 45 minutes? If so, explain how. If not, explain why.