Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. See "Guidelines and advice" on the course webpage for more information.

1. [Ch 9] Each of the following statements is either true or false. If a statement is true, then prove it. If a statement is false, then disprove it.
   (a) If $x, y \in \mathbb{R}$, then $|x + y| = |x| + |y|$.
   (b) There exist prime numbers p and q for which $p - q = 97$.
   (c) If $a, b, c \in \mathbb{Z}$, then at least one of $a - b$, $a + c$, and $b - c$ is even.
2. Use a "no smallest counterexample" argument to prove that for each $n \in \mathbb{N}$, we have $\displaystyle\sum_{k=1}^{n} \frac{k}{(k+1)!} = 1 - \frac{1}{(n+1)!}$.
3. Use induction to prove the following.
   (a) For each $n \in \mathbb{N}$, we have $\sum_{i=1}^{n}(8i - 5) = 4n^2 - n$.
   (b) For each non-negative integer n, we have $9 \mid 4^{3n} + 8$.
   (c) If $n \in \mathbb{N}$, then $2^n + 3^n \leq 5^n$.