

Directions:

1. Write your name with one character in each box below.
2. Show all work. No credit for answers without work.

1. [4 parts, 1 point each] Compute the derivatives of the following functions.

(a) $f(x) = \cos(1)$

↑
constant function

$$\frac{d}{dx} [\cos(1)] = 0$$

(b) $g(t) = \frac{2t+2}{t+2}$

$$g'(t) = \frac{(t+2) \frac{d}{dt} [2t+2] - (2t+2) \frac{d}{dt} [t+2]}{(t+2)^2}$$

$$= \frac{2(t+2) - (2t+2)}{(t+2)^2} = \boxed{\frac{2}{(t+2)^2}}$$

(c) $u(z) = \tan^3(z^2)$

$$\begin{aligned} \frac{du}{dz} &= 3 \tan^2(z^2) \cdot \frac{d}{dz} [\tan(z^2)] \\ &= 3 \tan^2(z^2) \cdot \sec^2(z^2) \cdot \frac{d}{dz} [z^2] \\ &= 3 \tan^2(z^2) \sec^2(z^2) \cdot 2z \\ &= \boxed{6z \tan^2(z^2) \sec^2(z^2)} \end{aligned}$$

(d) $h(y) = \ln(\sqrt{y})$

$$= \ln(y^{1/2}) = \frac{1}{2} \ln y$$

$$\frac{d}{dy} \left[\frac{1}{2} \ln y \right] = \frac{1}{2} \cdot \frac{1}{y} = \boxed{\frac{1}{2y}}$$

2. [4 parts, 1 point each] Compute the following indefinite integrals.

(a) $\int \frac{1}{2z} dz = \frac{1}{2} \int \frac{1}{z} dz$

$$= \boxed{\frac{1}{2} \ln |z| + C}$$

(b) $\int \sec^2 x dx$

$$= \boxed{\tan x + C}$$

(c) $\int \csc x \cot x dx = \boxed{-\csc x + C}$

(d) $\int (y+2)(y^2-1) dy = \int y^3 - y + 2y^2 - 2 dy$

$$= \int y^3 + 2y^2 - y - 2 dy =$$

$$= \boxed{\frac{1}{4} y^4 + \frac{2}{3} y^3 - \frac{1}{2} y^2 - 2y + C}$$

3. [2 points] Find the number b so that the area under the curve $y = x^2$ from $x = 1$ to $x = b$ is exactly 1.

Set (Area under the curve) = 1 and solve for b :

$$\int_1^b x^2 dx = 1$$

$$\left. \frac{1}{3} x^3 \right|_1^b = 1$$

$$\frac{1}{3} b^3 - \frac{1}{3} = 1$$

$$\frac{1}{3} (b^3 - 1) = 1$$

$$b^3 - 1 = 3$$

$$b^3 = 4$$

$$\boxed{b = 4^{1/3}}$$