Name: _____

Directions: Show all work. No credit for answers without work.

1. **[3 points]** Determine if the following vectors form a basis for $\mathbb{R}^4$.

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -6 \\ -3 \\ 6 \end{bmatrix}, \begin{bmatrix} -8 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

2. **[3 points]** Given the matrix A and an echelon form of A, find a basis for $\text{Col}(A)$ and $\text{Nul}(A)$.

$$A = \begin{bmatrix} 75 & 383 & -281 & 32 & 329 \\ 10 & 51 & -37 & 4 & 44 \\ 38 & 194 & -142 & 16 & 167 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -3 & 0 & 6 \\ 0 & 1 & -7 & 4 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. **[2 points]** Let H be the set of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4$ such that $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 + 2x_2 + 3x_3 + 4x_4 = 0$. Find a basis for H.

4. **[2 parts, 1 point each]** Subspaces.

   (a) Let H_1 and H_2 be subspaces of $\mathbb{R}^n$. Prove that $H_1 \cap H_2$ is also a subspace of $\mathbb{R}^n$.

   (b) Give an example of subspaces H_1 and H_2 of $\mathbb{R}^2$ such that $H_1 \cup H_2$ is not a subspace.