Name: ____________________

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** Decide whether the given transformation is linear. If the transformation is is linear, give the standard matrix. If the transformation is not linear, then explain why.

   (a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ |x_1 + x_2| \end{bmatrix}$

   (b) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} e^3 x_1 + \tan(\pi/5) x_2 \\ -x_1 \end{bmatrix}.$

2. **[1 point]** Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform, and let $\mathbf{v}_1, \dots \mathbf{v}_p$ be vectors in $\mathbb{R}^n$. Show that if $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is a linearly dependent set, then $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_p)\}$ is linearly dependent.

3. **[2 parts, 2 points each]** Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transform, let $\mathbf{u} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. We know that T maps $\mathbf{u}$ to $\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ and T maps $\mathbf{v}$ to $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$.

   (a) Find the image of $2\mathbf{u} - \mathbf{v}$ under T.

   (b) If possible, then find $T(\mathbf{w})$, where $\mathbf{w} = \begin{bmatrix} 1 \\ -19 \end{bmatrix}$. If not possible, then explain why not.

4. **[1 point]** Give a simple example of a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ such that the range of T is $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + x_2 + x_3 = 0 \right\}$.