

Name: _____

Directions: Show all work. No credit for answers without work.

1. **[3 points]** Find a basis for the eigenspace corresponding to the given eigenvalue.

$$\begin{bmatrix} 1 & 0 & -3 \\ -6 & -2 & 6 \\ 0 & 0 & -2 \end{bmatrix}, \lambda = -2$$

2. **[2 parts, 2 points each]** Find the characteristic equation and eigenvalues with multiplicities of the following matrices.

(a) $\begin{bmatrix} -6 & 5 \\ -10 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} -8 & 10 & 0 \\ -5 & 7 & 0 \\ 10 & -10 & 2 \end{bmatrix}$

3. [1 point] Let A be an $n \times n$ matrix. Prove that if A and I_n are similar, then $A = I_n$.
4. [4 parts, 0.5 points each] True/False. In the following, A and B are $n \times n$ matrices. Justify your answers.
- (a) If A and B have the same set of eigenvectors, then A and B are similar.
 - (b) If A and B are similar, then A and B have the same set of eigenvectors.
 - (c) If A and B have the same characteristic polynomial, then A and B are similar.
 - (d) If A and B are similar, then A and B have the same characteristic polynomial.