Name: _____

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, where $\mathbf{b}_1 = \begin{bmatrix} 5 \\ -1 \\ 4 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{b}_3 = \begin{bmatrix} 2 \\ 0 \\ 3 \\ 1 \end{bmatrix}$. Given $\mathbf{x} = \begin{bmatrix} 3 \\ 12 \\ 14 \\ 10 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{B}}$ if possible.

2. **[2 points]** Let A be a 5×8 matrix.

   (a) What are the possible values for the dimension of the null space of A?

   (b) Suppose that the transform T given by $T(\mathbf{x}) = A\mathbf{x}$ is onto/surjective. Now what are the possible values for the dimension of the null space of A?

3. Compute the determinant of the following matrices.

(a) **[1 point]** $\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$

(b) **[1 point]** $\begin{bmatrix} 2 & -1 & 5 \\ 0 & 1 & -2 \\ 1 & 7 & -2 \end{bmatrix}$

(c) **[2 points]** $\begin{bmatrix} 2 & 6 & 0 & -5 \\ 4 & 1 & 3 & 8 \\ 0 & 5 & 0 & 0 \\ 3 & -2 & 0 & 1 \end{bmatrix}$

(d) **[2 points]** $\begin{bmatrix} 1 & 3 & 1 & 5 \\ -2 & -4 & -3 & -6 \\ 1 & 3 & 2 & 8 \\ 3 & 9 & 3 & 12 \end{bmatrix}$ (Hint: use row reduction)