Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [2.8.{15-17}] Determine which sets below are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify each answer.

   (a) $\begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 10 \\ -3 \end{bmatrix}$

   (b) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$

2. [2.8.{23-25}] Given a matrix A and an echelon form of A, find a basis for Col(A) and Nul(A).

   (a) $A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 8 & 0 & 5 \\ 0 & 2 & 5 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. [2.8.{21,22}] True/False. Justify your answers.

   (a) A subspace of $\mathbb{R}^n$ is any set H such that (i) the zero vector is in H, (ii) $\mathbf{u}, \mathbf{v}$, and $\mathbf{u} + \mathbf{v}$ are in H, and (iii) c is a scalar and $c\mathbf{u}$ is in H.

   (b) If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are in $\mathbb{R}^n$, then Span$\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is the same as the column space of the matrix $[\mathbf{v}_1 \cdots \mathbf{v}_p]$.

   (c) The set of all solutions of a system of m homogeneous equations in n unknowns is a subspace of $\mathbb{R}^m$.

   (d) The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

   (e) Row operations do not affect linear dependence relations among the columns of a matrix.

   (f) A subset H of $\mathbb{R}^n$ is a subspace if the zero vector is in H.

   (g) Given vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$, the set of all linear combinations of these vectors is a subspace in $\mathbb{R}^n$.

   (h) The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.

   (i) The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{b}$.

   (j) If B is an echelon form of a matrix A, then the pivot columns of B form a basis for A.