Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the "General Guidelines and Advice" on the homework page for more details.

1. For $n \geq 4$, determine the maximum number of edges in an n-vertex graph G in which every pair of cycles shares a common edge.

2. For $k > 0$, let G be a k-regular graph of even order that remains connected whenever $k-2$ edges are deleted. Prove that G has a 1-factor.

3. Let G be a graph, and let H be the block-point forest of G. Recall: H is the $(\mathcal{B}, S)$-bigraph, where $\mathcal{B}$ is the set of blocks, S is the set of cut vertices, where a block B is adjacent to a cut-vertex u if and only if $u \in V(B)$. Prove that H is a forest in which every leaf belongs to $\mathcal{B}$.

4. Let v be a vertex of a 2-connected graph G. Prove that v has distinct neighbors u_1 and u_2 such that $G - v - u_1$ and $G - v - u_2$ are both connected. Hint: consider the block-point forest of $G - v$.

5. Let x and y be vertices in a 3-connected graph G. Show that there is an induced xy-path P such that $G - V(P)$ is connected.

6. Let G be a $2k$-edge-connected graph with at most two vertices of odd degree. Prove that G has a k-edge-connected orientation. (Recall that a digraph D is k-edge-connected if $|[S, \overline{S}]| \geq k$ when S is a nonempty proper subset of $V(D)$. Here, the directed cut $[S, \overline{S}]$ is the set of all edges from vertices in S to vertices outside S.)