Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. A non-zero vector $\mathbf{v}$ is a *generalized eigenvector* of A associated with eigenvalue λ if $\mathbf{v}$ is in the null space of $(A - \lambda I)^k$ for some positive integer k. Prove that if $\mathbf{v}$ is a generalized eigenvector of A associated with λ and $A = P^{-1}BP$, then $P\mathbf{v}$ is a generalized eigenvector of B.

2. [5.9.{3,13}] On any given day, a student is either healthy or ill. Of the students who are healthy today, 95% will be healthy tomorrow. Of the students who are ill today, 55% will still be ill tomorrow.

   (a) What is the stochastic matrix P that models this situation?

   (b) Suppose 20% of the students are ill on Monday. What fraction or percentage of the students are likely to be ill on Tuesday? On Wednesday?

   (c) If a student is well today, what is the probability that he or she will be well two days from now?

   (d) Find the steady-state vector for P.

   (e) What is the probability that after many days a specific student is ill? Does it matter if that person is ill today?

3. [5.9.{15-20}] True/False. In the following, P is an $n \times n$ stochastic matrix. Justify each answer.

   (a) The steady state vector is an eigenvector of P.

   (b) Every eigenvector of P is a steady state vector.

   (c) The all ones vector is an eigenvector of P^T.

   (d) All stochastic matrices are regular.