Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [5.3.4] Given the factorization $A = PDP^{-1}$ below, find a formula for A^k where k is a non-negative integer.

$$\begin{bmatrix} -6 & 8 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

2. [5.3.{7-20}] Diagonalize the following matrices if possible. That is, for each diagonalizable matrix A below, construct an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. (There is no need to compute P^{-1} explicitly.) For each matrix A below that is not diagonalizable, explain why not.

   (a) $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 4 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

   (e) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

3. [5.3.{21-28}] True/False. In the following, A, P, and D are $(n \times n)$ matrices. Justify your answers.

   (a) A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P.

   (b) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable.

   (c) A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.

   (d) If A is diagonalizable, then A is invertible.

   (e) A is diagonalizable if A has n eigenvectors.

   (f) If A is diagonalizable, then A has n distinct eigenvalues.

   (g) If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.

   (h) If A is invertible, then A is diagonalizable.

4. [5.3.31] A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that A is not diagonalizable? Justify your answer.

5. Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, and let λ_1 and λ_2 be scalars. Construct a (2×2)-matrix A having eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$ with respective eigenvalues λ_1 and λ_2. (Here, the entries of A will depend on λ_1 and λ_2.)

6. [5.8.8] Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ and let $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Execute the power method to generate $\mathbf{x}_k$ and μ_k for $k = 0, \ldots, 4$, keeping 3 decimal places. What is the estimated eigenvalue/eigenvector pair?