Name: ______________________________

Directions: Show all work. Answers without work generally do not earn points. Unless stated otherwise, answers may be left in terms of factorials and binomial coefficients.

1. [**4 points**] A restaurant offers 7 different sandwiches, 5 sides, 6 soups, and 3 desserts. The lunch special consists of a sandwich, a choice of 1 side or a soup (but not both), and a dessert. How many ways are there to order the lunch special?

2. [**4 points**] How many ways are there to distribute 35 identical gold coins among 8 people?

3. [**2 parts, 4 points each**] How many 5-digit ATM pins:

   (a) contain only even digits?

   (b) contain at least one odd digit?

4. **[3 parts, 4 points each]** A game system has 4 buttons in different colors: red, green, blue, and yellow. The buttons must be pressed one at a time, in some order. To win the game, each button must be pressed twice. How many ways are there to win:

   (a) with no additional restrictions?

   (b) if the green presses must occur consecutively?

   (c) if both red presses must occur before both blue presses?

5. **[3 parts, 4 points each]** *Word arrangements.* How many ways are there to arrange the letters of 'APPROPRIATE':

   (a) with no additional restrictions.

   (b) with no two consecutive P's.

   (c) with all P's separated by at least 2 letters. (So PAAPROPRITE counts but PAPROPRIATE does not.

6. *Poker hands.* Recall that a deck of cards has 4 suits (clubs, diamonds, hearts, and spades) and 13 ranks (ace, 2 through 10, jack, queen, and king). There are 52 cards (one for each suit/rank pair). A poker hand is a set of 5 cards (order does not matter). The *face cards* are the cards whose rank is jack, queen, or king.

   (a) [**4 points**] How many poker hands have no face cards?

   (b) [**1 point**] What are the odds of being dealt a poker hand with no face cards? Round your answer to the nearest decimal percentage of the form $xx.xx\%$.

   (c) [**4 points**] How many hands have 3 cards in one suit and 2 cards in a different suit?

   (d) [**4 points**] How many hands have all distinct ranks and at least 1 card in each suit?

7. **[5 parts, 3 points each]** Count the non-negative integer solutions to $x_1 + \cdots + x_5 = 40$:

   (a) with no additional restrictions.

   (b) with $x_i \geq 3$ for all i.

   (c) with $x_2 \leq 18$

   (d) with $x_4 = 20$ and $x_5 = 10$

   (e) with $x_4 = 20$ or $x_5 = 10$ (or both)

8. **[3 parts, 4 points each]** Find the coefficient:

   (a) of x^4y^5 in $(x+y)^9$

   (b) of $x^3y^4z^5$ in $(x+y+z)^{12}$

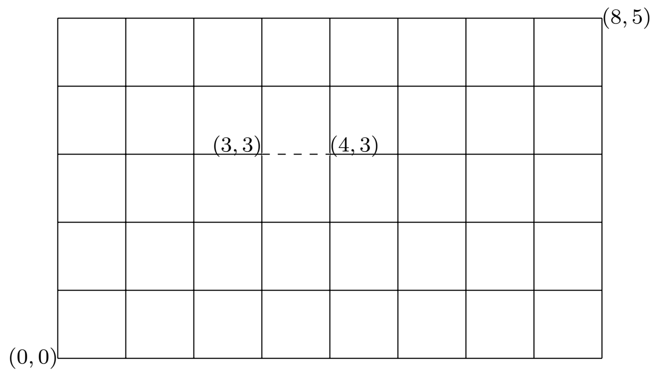
   (c) of $x^5y^5z^5$ in $(2x-y+3z)^{15}$

9. **[2 parts, 4 points each]** Give simple formulas for the following sums:

   (a) $\sum_{k=1}^{n} k$

   (b) $\sum_{k=0}^{n} \binom{n}{k} 3^k$

10. **[2 parts, 4 points each]** *Lattice Paths.* Recall that a step in a lattice path increases one of the coordinates by 1.



(a) How many lattice paths are there from $(0,0)$ to $(8,5)$?

(b) How many of these paths avoid the segment from $(3,3)$ to $(4,3)$ (depicted above with a dashed line segment)?

11. **[4 points]** *Lattice paths in 3 dimensions.* In 3 dimensions, a step in a lattice path moves from (x, y, z) to one of the following points: $(x+1, y, z)$, $(x, y+1, z)$, $(x, y, z+1)$. How many lattice paths are there from $(0,0,0)$ to (n,n,n)? Hint: apply the method that allowed us to count 2-dimensional lattice paths.