Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Let $\Sigma = \{0, 1\}$. Determine $|\Sigma^5|$.

$$|\Sigma^5| = 2^5 = \boxed{32}$$

2. Let $\Sigma = \{0, 1\}$ and let A be the language defined recursively as follows:

   (1) $\lambda \in A$.

   (2) If $x \in A$, then $0x \in A$.

   (3) If $x \in A$, then $x1 \in A$.

   (a) **[2 points]** List all strings in A of length at most 2.

$$\boxed{\lambda, 0, 1, 00, 01, 11}$$

   (b) **[1 point]** Give a simple, non-recursive description of A.

$$A = \{0^r 1^s : r, s \geq 0\}$$

or A is the set of strings with some zeros followed by some ones.

or A is the set of strings with all zeros before all ones.

or A is the set of strings that do not contain 10 as a substring.

   (c) **[1 point]** Give a formula for the number of strings in A of length n.

We want $|\{0^k 1^{n-k} : 0 \leq k \leq n\}|$.

This equals $\boxed{n+1}$.

3. **[4 parts, 1 point each]** Let $\Sigma = \{0, 1, 2\}$. Let A be the language of all strings over Σ with an equal number of 0's and 1's, and let B be the language of all strings over Σ with an equal number of 1's and 2's. For example, if $w = 1202112$, then the number of occurrences of 0, 1, and 2 in w is 1, 3, and 3 respectively. Consequently $w \notin A$ and $w \in B$.

   (a) Give a simple, English description of the language $A \cap B$.

$A \cap B$ is the language of strings that have equal number of 0's, 1's, and 2's.

   (b) Determine the number of strings in $A \cap B$ of length 10.

Every string in $A \cap B$ has a length that is a multiple of 3. Since 10 is not a multiple of 3, the number of these strings is $\boxed{0}$.

   (c) True or false: if x and y are both in A, then $xy \in A$. $\boxed{\text{True}}$

   (d) Give an example of a string w over Σ that has length 2 such that $w \in AB$ and $w \notin BA$.

The only such string is $\boxed{20}$.

Since $2 \notin A$ and $0 \in B$, we have $20 \in AB$.

It is easy to check $20 \notin BA$, by examining all 3 ways of splitting 20 into two strings.