Directions: Show all work. No credit for answers without work.

1. **[2 points]** Find $\mathcal{P}(\{\varnothing\})$.

$$\mathcal{P}(\{\varnothing\}) = \boxed{\{\varnothing, \{\varnothing\}\}}$$

2. **[2 parts, 1 point each]** Let $A = \{(1,2), (5,\{6,7\},8), 9, \{10,11\}\}$ and let $B = \{(2,1), 9, 10, 11, \{\cancel{10}, 11\}\}$. 11 10

   (a) Determine $|A|$ and $|B|$.

   $|A| = 4$

   $|B| = 5$

   (b) Determine $A \cap B$.

   $A \cap B = \{9, \{10, 11\}\}$

3. **[2 parts, 1 point each]** True or false (write the whole word):

   (a) $(5, \{3,1\}, 8) = (5, \{1,3\}, 8)$ True

   (b) $\{5, \{3,1\}, 8\} = \{5, \{1,3\}, 8\}$ True

4. Let $U = \{x \in \mathbb{Z} \mid 1 \leq x \leq 3\} \cup \{x \in \mathbb{Z} \mid -3 \leq x \leq -1\}$.

(a) **[1 point]** List the elements of U. What is $|U|$?

$U = \{-3, -2, -1, 1, 2, 3\}$. $|U| = 6$

(b) **[1 point]** Let A be the set of all subsets of U that do not contain a pair of integers with sum zero. For example, $\{-2, 1\}$ and $\varnothing$ are members of A but $\{-1, 1, 2\} \notin A$ because $-1 + 1 = 0$. Give an example of a set $S \in A$ such that $|S| = 3$.

Many correct answers; for example, $S = \{-3, 2, -1\}$.

(c) **[2 points]** Let $B = \{(x_1, x_2, x_3) \mid x_i \in \{-, 0, +\}$ for each $i\}$. For example, $(0,0,0)$, $(+,0,+)$ and $(-,+,+)$ are all elements of B. (1) Describe a bijective correspondence between A and B. (2) What is $|A|$?

(1). Given a set $S \in A$, we pair S with

$(x_1, x_2, x_3) \in B$ where

$$x_i = \begin{cases} - & \text{if } -i \in A \\ + & \text{if } +i \in A \\ 0 & \text{if } -i, +i \notin A. \end{cases}$$

For example,

$$\{-2, 1\} \longleftrightarrow (+, -, 0)$$
$$\{-3, 2, -1\} \longleftrightarrow (-, +, -).$$
$$\varnothing \longleftrightarrow (0, 0, 0)$$

(2). We have $|A| = |B| = 3^3 = \boxed{27}$, since ~~a~~ an element of B is formed by making ~~B~~ choices in 3 stages, each with 3 options:

$$(x_1, x_2, x_3).$$
$$3 \times 3 \times 3 = 3^3$$