Name: Solutions

Directions: Show all work. No credit for answers without work.

1. Poker hands.

(a) **[2 points]** How many poker hands have 3 clubs, 1 heart, and 1 diamond?

1. Choose ranks for clubs $\binom{13}{3}$
2. Choose a heart $\binom{13}{1}$
3. Choose a diamond $\binom{13}{1}$

So total # = $\binom{13}{3}\cdot\binom{13}{1}\cdot\binom{13}{1} = \boxed{48,334}$

(b) **[2 points]** How many poker hands have 5 cards of distinct ranks? For example, 3H 4H 5H 6H 7H counts but 3H 3S 4D 5C 6C does not.

1. Choose 5 distinct ranks $\binom{13}{5}$
2. Choose a suit for the ~~first~~ lowest rank 4
3. " " " " " #2 rank 4

⋮

6. " " " " " highest rank 4

total # = $\binom{13}{5}\cdot 4^5 = \boxed{1,317,888}$

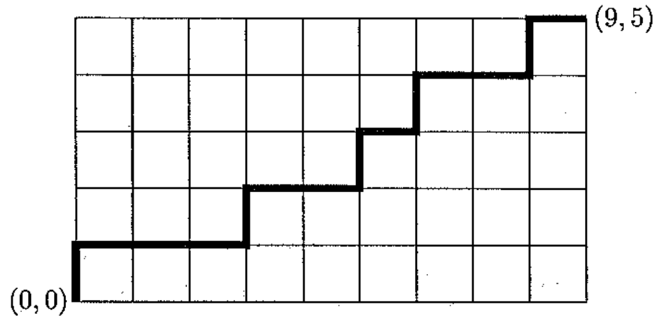
(c) **[2 points]** In a standard deck of cards, the hearts and diamonds are red and the clubs and spades are black. How many poker hands have at least one card in each color?

(# hands) = (# all red) + (# all black) + (# mixed)

$\binom{52}{5} = \binom{26}{5} + \binom{26}{5} + x$

$x = \binom{52}{5} - 2\binom{26}{5} = \boxed{2,467,400}$

2. Lattice paths from $(0,0)$ to $(9,5)$. Recall that each step of a lattice path increases one of the coordinates by 1; geometrically, we either move one unit in the horizontal direction or 1 unit in the vertical direction.



(a) **[2 points]** How many lattice paths are there from $(0,0)$ to $(9,5)$?

Arrange 9 R's and 5 U's in some order:

$$\frac{(9+5)!}{(9!)(5!)} = \binom{14}{5} = \boxed{2{,}002}$$

(b) **[1 point]** How many lattice paths from $(0,0)$ to $(9,5)$ move in the horizontal direction for their last step? (One such path is highlighted in bold above.)

Fix R at end; arrange remaining 8 R's and 5 U's:

$$\frac{(8+5)!}{(8!)(5!)} = \binom{13}{5} = \boxed{1{,}287}$$

(c) **[1 point]** How many lattice paths from $(0,0)$ to $(9,5)$ never move in the vertical direction twice in a row? (One such path is highlighted in bold above.)

Write down 9 R's: R R R R R R R R R
~~Insert~~ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

Choose ~~the~~ 5 of the available 10 spaces to in sert the U's.

So # lattice paths with no consecutive U's is $\binom{10}{5}$ or $\boxed{252}$.