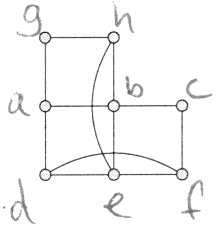
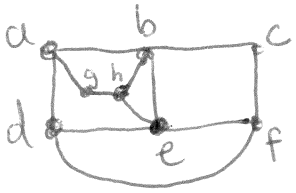
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. Either give a planar drawing or show that the graph contains a subdivision of K_5 or $K_{3,3}$.



Planar:



2. Prove that every planar graph has a vertex of degree at most 5.

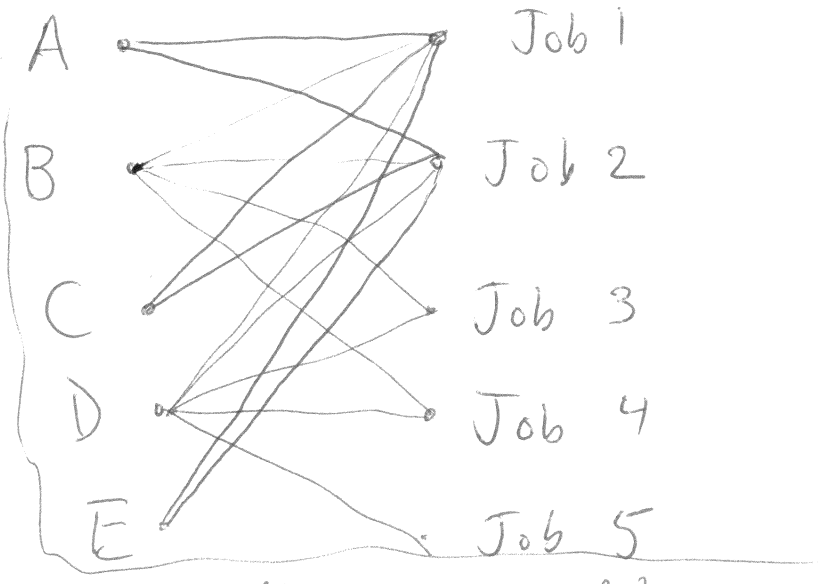
Let G be an n-vertex planar graph.

Then $2|E(G)| = \sum_{v \in V(G)} d(v)$ and $|E(G)| \leq 3n-6$.

So $\sum_{v \in V(G)} d(v) = 2|E(G)| \leq 2(3n-6) = 6n-12$.

If each vertex in G has degree at least 6, then the degree sum would be at least $6n$. Since $6n \leq \sum_{v \in V(G)} d(v) \leq 6n-12$ is not possible, some vertex has degree less than 6.

3. Five people need to do a total of 5 jobs, and no person can do more than one job. Alice can do 2 of the tasks, Bob can do all 5, Calvin can do 2, Danielle can do 4, and Evan can do 2. Give an example showing that it may not be possible to complete all 5 tasks.

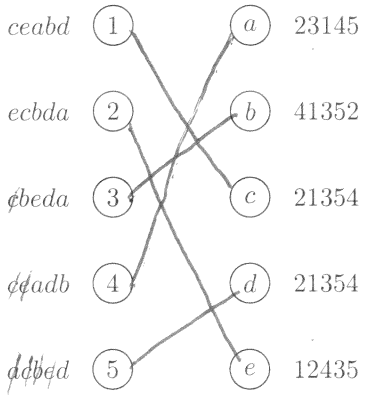


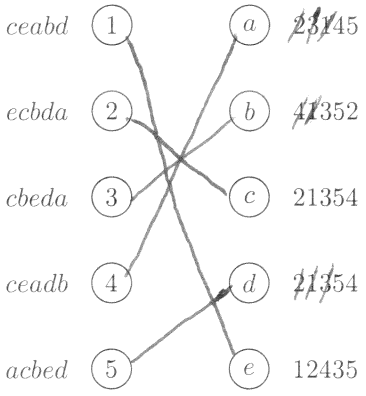
Note that A, C, E can only complete the same two jobs.

So for $S = \{A, C, E\}$, we have

$3 = |S| > |N(S)| = 2$, and Hall's condition fails.

4. Given a set $\{1, 2, 3, 4, 5\}$ of men and a set $\{a, b, c, d, e\}$ of women with the following preference lists, find the stable matching resulting when (a) men propose to women, and (b) when women propose to men.





Men propose

women propose